LEVEL II

2

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

A METHODOLOGY TO FIND OVERALL SYSTEM EFFECTIVENESS IN A MULTICRITERION ENVIRONMENT USING SURFACE TO AIR MISSILE WEAPON SYSTEMS AS AN EXAMPLE

by

Knut O. Flaathen

September 1981

Thesis Advisor: G. F. Lindsay

Approved for public release; distribution unlimited

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A209	3. RECIPIENT'S CATALOG NUMBER 549
4. TITLE (and Subtitle) A Methodology to Find Overall Effectiveness in a Multicriterion Environment Using Surface to Air Missile Weapon Systems as an Example		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; September 1981
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Knut O. Flaathen		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE September 1981
		13. NUMBER OF PAGES 138
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Overall system effectiveness, multicriterion environment, judgement modeling, constant sum scaling method, functional relationship

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Finding overall system effectiveness from a multicriterion environment using SAM weapon systems as an example, is the purpose of this thesis. SAM weapon systems were rated by four groups of experiences individuals, and judged overall system effectiveness for each system was calculated using the Constant Sum Scaling Method. Multiple regression analysis was then used to establish a functional relationship between overall system effectiveness and

DD FORM 1473 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE
(Page 1) S/N 0102-014-6601

251450 alk

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) weapon characteristics (including missile price). It was concluded that there were no significant differences among the judged results in the four groups nor between judged and functional overall system effectiveness.

Accession For
NTIS GRA&I X
Dist
A

Approved for public release; distribution unlimited

A Methodology to Find Overall System Effectiveness in a Multicriterion Environment Using Surface to Air Missile Weapon Systems as an Example

by

Knut O. Flaathen
Lieutenant Commander, Royal Norwegian Navy
Graduate, Royal Norwegian Naval Academy, 1970

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1981

Author ______________________

Approved by: Glenn F. Lindsay
Thesis Advisor

Second Reader

Chairman, Department of Operations Research

W M Woods
Dean of Information and Policy Sciences

ABSTRACT

Finding overall system effectiveness from a multicriterion environment using SAM weapon systems as an example, is the purpose of this thesis. SAM weapon systems were rated by four groups of experienced individuals, and judged overall system effectiveness for each system was calculated using the Constant Sum Scaling Method. Multiple regression analysis was then used to establish a functional relationship between overall system effectiveness and weapon characteristics (including missile price). It was concluded that there were no significant differences among the judged results in the four groups, nor between judged and functional overall system effectiveness.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

I. INTRODUCTION

A measure of effectiveness (MOE) is a correlate, an estimator, or a predictor of true value. It is used to find out how well an existing system works, or to find out what an existing system is worth compared to other similar systems. A MOE can be used to make an existing system work better, or to design, select, and prepare to operate future systems so that they will achieve a higher performance. A MOE should be operational, measurable, analytically tractable, and able to support decision making [1].

The MOE of a weapon system is an important, if not one of the most important aspects of military planning. "Which system is most effective?", "how much better is one weapon system than another among similar systems?", "what effect will a change in a major characteristic of the system have on the overall MOE of the system?", are questions that have to be answered before any final decision can be taken about which weapon system to buy.

In this paper Surface to Air Missile (SAM) weapon systems are chosen to illustrate one methodology used to answer such questions. A structured relationship between MOE's obtained from military experts' judgments, and major system characteristics will be developed, so that experts' judgments will not necessarily be required when the performance of similar systems are to be assessed in the future.

Chapter II will give the research approach (and what's unusual about it). Chapter III will cover the concept and the general experimental procedure. The chapter will discuss the choice of the major SAM characteristics, and how necessary data was collected. Selection and grouping of judges will also be outlined. The content of Chapter IV is an introduction to the Constant Sum Scaling Method, and the use of the method to compute the overall system effectiveness for each weapon system, within each selected group of judges.

A functional relationship between the system effectiveness and the system characteristics will then be established in Chapter V using multiple linear and nonlinear regression analysis. Major conclusions, observations, and recommendations will be given in the final chapter.

II. RESEARCH APPROACH

An MOE is normally used together with a concept or model of a system of operations (characteristics for SAM weapon systems in this study). Combining individual MOE's for each operation (characteristic) into an overall system effectiveness is not a trivial problem. The usual approach is to find some linear or nonlinear combination of the individual MOE's that will give an overall MOE for the entire system of operations. The equation obtained from the best combination will give an estimate of the overall system effectiveness. There is however no way the obtained estimator can be tested because the true overall system effectiveness is indeed unknown.

A different approach, that attempts to find an equation that

(i) tends to reflect that way decision makers are thinking, and

(ii) can be tested,

is the main purpose of this paper. In Chapter IV an overall judged system effectiveness value will be established for each of seven SAM systems, independently of any linear or nonlinear combination of individual MOE's. Then in Chapter V, these judged overall system effectiveness values will be compared with least-squared error models of the individual

MOE's (characteristics). The difference between the two independently obtained overall system effectivenesses is then reflected in the least-squared error ($SE = (S - \hat{S})^2$), which is a good measure of the accuracy of the candidate model. A methodology has thus been established that allows testing of the overall system effectiveness models. This area of analysis is found under various titles, but is most often referred to as Policy Capturing [2]. For the purpose of this paper, judgement modeling will probably be a more consistent terminology.

It must be emphasized that this paper will estimate the overall system effectiveness of SAM weapon systems by measuring and judging only selected operational characteristics and missile prices. Other elements of combat that are of equal or greater importance will not be reflected in this research. It should thus be recognized that the applied methodology has substantial limitations.

III. EXPERIMENTAL DESIGN

This chapter describes the general concept of a functional relationship between independent and dependent variables, or in other words, between individual weapon system characteristics (MOE's) and judged overall system effectiveness, respectively. Another purpose of the chapter is to demonstrate how data was collected, and further to discuss selection of weapon systems and characteristics, using SAM weapon systems as example.

A. CONCEPT

One problem to be solved in this paper is how to find a function that can estimate one set of dependent data (overall system effectiveness) from another independent set of data (system characteristics). This concept is notationally expressed in Figure 1, or if expressed in matrix notation as:

$$\hat{\underset{\sim}{S}} = (\underset{\sim}{X}_1 , \underset{\sim}{X}_2 , \text{---} , \underset{\sim}{X}_m) . \qquad (1)$$

Instance	Estimated Values For Independent Variables	Function	Independent Variables Known Values		
1	$\hat{S}_1$		X_{11}	X_{12}	$---X_{1m}$
2	$\hat{S}_2$		X_{21}	X_{22}	$---X_{2m}$
$\vdots$	$\vdots$	F		$\vdots$	$\vdots$
n-1	$\hat{S}_{n-1}$		$X_{n-1,1}$	$X_{n-1,2}$	$---X_{n-1,m}$
n	$\hat{S}_n$		X_{n1}	X_{n2}	$---X_{nm}$

Figure 1
Functional Relationship [3: p. 53]

The above model (relationship) has n systems or instances, and thus n overall system effectivenesses have to be estimated. Mathematically each estimated value would then be noted:

$$\hat{S}_i = F\,(X_{i1}, X_{i2}, --- , X_{im});\ i = 1,2, ---,n. \qquad (2)$$

B. GENERAL OUTLINE

Figure 2 illustrates how the experimental procedure is divided into three separate sections. A detailed discussion of Section I will be covered in this general outline, while Sections II and III (scaling to determine overall system effectiveness and determination of the functional relationship between overall system effectiveness and system characteristics) will be discussed in Chapters IV and V respectively.

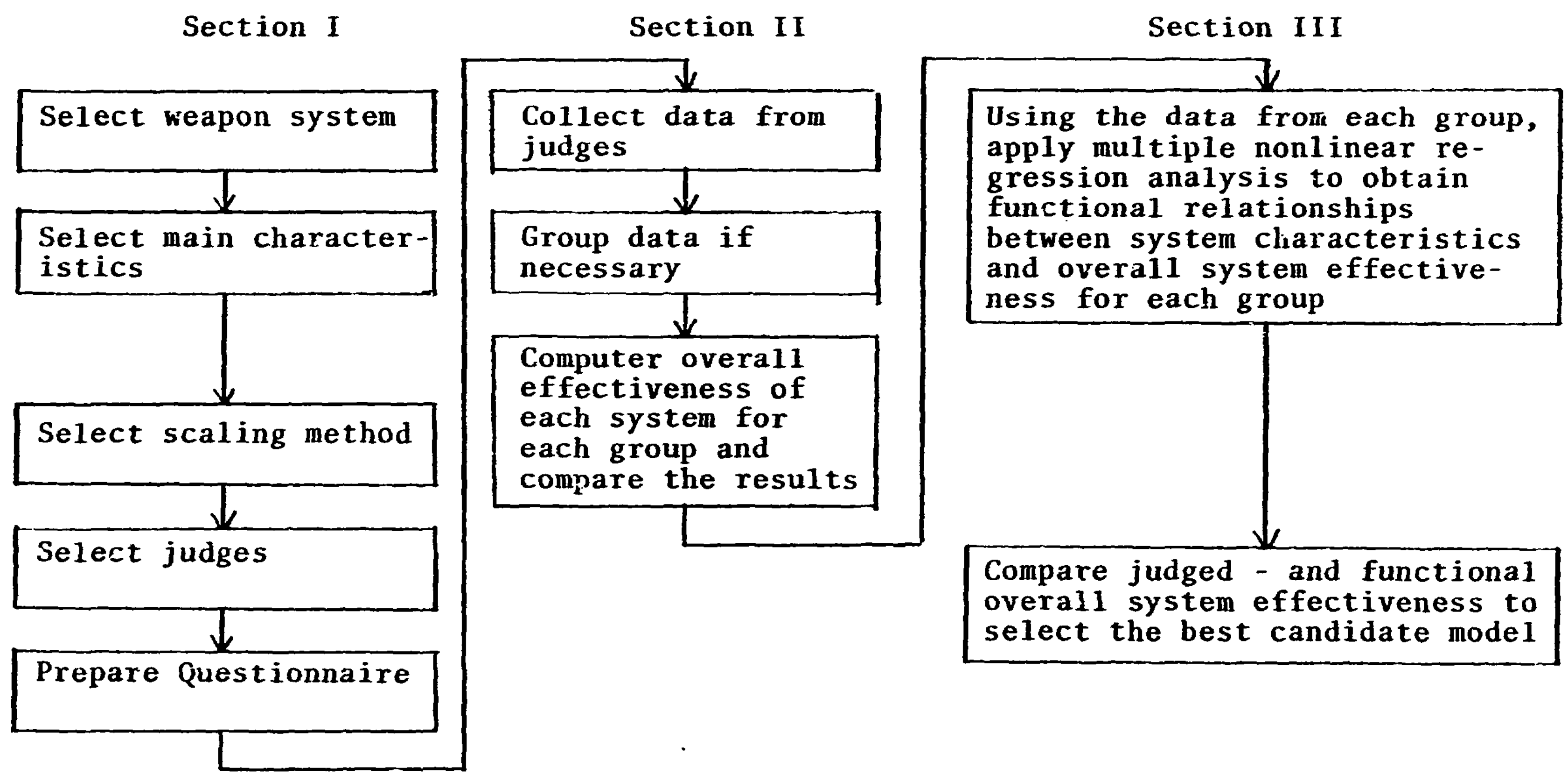


Figure 2

Block Diagram Representing the Study

1. Selection of a Weapon System and Its Major Characteristics

SAM weapon systems were chosen to illustrate the methodology of finding overall system effectiveness of weapon systems. In order to avoid using classified data, and further to avoid judges having certain preferences to well-known systems that unconsciously could change their judging, seven fictitious SAM weapon systems (A - G) were designed. Real-life systems were thoroughly studies to make the designed systems as realistic as possible. The primary operational mission was chosen to be point-to-point defense with area defense as a secondary mission. Selection of weapon system characteristics proved to be more complex than imagined. There are of course, a large variety of characteristics that affect the effectiveness of a weapon system. The fact however that some characteristics differ very little among different systems made the choice a little easier. These characteristics could be excluded because they would not make any significant changes in the analysis. Finally, the following four SAM weapon system characteristics were selected together with missile price:

X_1 : kill probability of a single shot

X_2 : reaction time (seconds from detection to missile launch)

X_3 : max effective range (in km)

X_4 : average missile speed (in mach)

X_5 : missile price (in 10,000 of dollars).

The operational aspects together with the purchase price of new missiles were considered as the most important semblance to this study, and were thus the main reason for the choice of the above characteristics. Other characteristics like mobility, missile guidance, and system maintainability are all important characteristics, but were considered less operationally significant. In addition, it would be difficult to obtain useful numerical values for each of them due to lack of standard measurements. The characteristic values describing the seven fictic SAM weapon system are shown in Table 1.

2. Selection of Scaling Method

Many scaling methods could be used to obtain system effectiveness by judges using data from Table 1. Numerical evaluation, ordinal, categorial judgement, or the Constant Sum Scaling Method could all be used. In this study it is however a question about judging how much better one system is than another. A ratio scale that can be used directly for comparison of the two systems is thus necessary. Judgments are further required on a rather high-level scale so only modest computational efforts (not time consuming) are needed. The number of systems to be compared is also rather moderate. Among those scaling methods available the Constant Sum Scaling Method seems to be one that fits the purpose of this study.

TABLE 1

Five Characteristic Values for Seven SAM Weapon Systems

FACTOR \ SYSTEM	A	B	C	D	E	F	G
Kill probability of single shot	0.90	0.75	0.85	0.70	0.65	0.80	0.[illegible]0
Reaction time (seconds from detection to missile launch)	6	30	10	8	30	12	15
Max Effective Range (in km)	9	12	15	8	22	18	26
Average missile speed (in Mach)	2.3	2.0	2.2	2.0	1.7	1.5	1.9
Missile-price (in 10,000 of $)	60	60	70	45	80	65	100

3. Selection of Judges

There appears to be no rule or standard for designating individuals as "experts". Officers with a good theoretical and practical background on SAM weapon systems proved to be hard to find. The chosen approach was therefore primarily to use the resources already available at the Naval Postgraduate School (NPS) in form of its officer students. A questionnaire was sent out to every Navy line officer with experience from surface-ships, and to every naval aviator. A total of 450 questionnaires were distributed at NPS and 112 were completed and returned. Of those, 51 were from line officers with experience from SAM weapon system. Later in the study these 51 responses will be referred to as Group 2. An additional 13 questionnaires were received from officer students having exceptionally good theoretical and practical background (Army, Air Force, or Naval officers with air defense (AD) billets, or with AD department head experience). Ten questionnaires were also completed and returned from the US Army Air Defense School at Fort Bliss, Texas, and 15 were received from the Royal Norwegian Air Defense Academy. All together this makes an additional group later referred to as Group 3, with 38 individual answers, considered to be the real experts' judgments. By combining all the obtained data, a fourth group with 137 completed questionnaires was established.

Having grouped the answers the above way, a wide variety of analytical judgments are covered. It was anticipated that Group 1, the naval line officers, would probably consider primarily the defensive aspect of the missile systems, and Group 2, the naval aviators, would equally probably consider primarily the offensive aspect. Group 3 would hopefully, being at a high level of experience, judge both the defensive and the offensive aspects.

4. Preparing the Questionnaire

Questionnaires employing the Constant Sum Scaling Method tent to be lengthy because n x (n - 1)/2 judgments have to be made (n being number of instances) [4]. In this study 21 pairs have to be judged. This requires a quick, easy and accurate method to compare two SAM weapon systems by their characteristics, and judge how much better one is than the other. Within each of the 21 pairs, the judges will be asked to make ratio scale judgments by splitting 100 points in term of the relative overall effectiveness of the two SAM weapon systems. For example: A 80 B 20 if the judge considers system A has four times the overall system effectiveness as system B, or: A 50 B 50 if the judge considers system A to be equally effective to system B. The questionnaire is displayed in Appendix A.

So far in this paper, seven SAM weapon systems with five characteristic values have been chosen as a data base. A scaling method has been selected, and a population of judges identified. Questionnaires have been sent, and answers have been collected. The next chapter will evaluate the information obtained from the judges, and establish the judged overall system effectiveness values for each weapon system within each of the four groups, using the Constant Sum Scaling Method.

IV. COMPUTATION OF SYSTEM EFFECTIVENESS

Having collected all necessary data, the next step is to compute the overall system effectiveness, and to compare the results obtained within each of the four groups.

A. CALCULATION OF THE OVERALL SYSTEM EFFECTIVENESS FOR EACH WEAPON SYSTEM WITHIN EACH GROUP USING THE CONSTANT SUM SCALING METHOD [5: pp. 105-116]

The Constant Sum Scaling Method is designed to scale a property having either a natural origin or an origin upon which judges agree [4]. The values sought and obtained in this study will be the system effectiveness values for each weapon system obtained from each group, labeled S_{ik}; $i = A, ---, G$; $k = 1, ---, 4$; such that for example S_{F3} will be system effectiveness obtained for Weapon System F from judgment Group 3. Each judge has been asked to make a ratio scale judgment by splitting 100 points within a pair of instances (weapon systems). If n were the number of instances, a total of $n \times (n-1)/2$ pairs had to be judged.

Let a_{ij} be the notation used to represent the number of points a judge gives to instance j when it is compared to instance i. For each judge the $n \times (n-1)/2$ responses can be arranged in a matrix A where cross diagonal elements sum to 100 and where all diagonal elements (representing instances compared to themselves) are 50. If there were p judges all together, a new matrix $\bar{A}$, being the average of all the individual response matrices, could be constructed with elements being

$$\bar{a}_{ij} = \frac{\sum_{i=1}^{p} a_i}{p} . \tag{3}$$

The next step is to compute a new n x n matrix W with elements $W_{ij} = \frac{\bar{a}_{ij}}{\bar{a}_{ji}}$. (4)

In W, cross-diagonal elements will be reciprocal to each other and diagonal elements will have the value 1. "Each element W_{ij} provides an estimate of the ratio of two of the scale values we are seeking, S_j and S_i, and we could write W_{ij} = estimate of

$$\frac{S_j}{S_i} = \frac{\text{Scale value of instance } j}{\text{Scale value of instance } i}$$ " [4: p. 3].

Since there are more estimates (21 W_{ij}'s) than there are instances (seven weapon systems) to be estimated the solution given in the W matrix will be overdetermined. One could for example compare systems A and B in (n - 1) different ways:

W_{AB} and $\frac{W_{iA}}{W_{iB}}$; i = C,D,E,F,G, where in general

$$W_{AB} \neq \frac{W_{iA}}{W_{iB}} .$$

To resolve this multiple estimate problem a least squares approach over the estimates may be used. If the estimation is perfect we would have

$$W_{ij} = \frac{S_j}{S_i} , \tag{5}$$

and by taking the natural log on both sides we get

$$\ln W_{ij} - (\ln S_j - \ln S_i) = 0. \tag{6}$$

To get as close as possible to this perfect solution we want $(\ln W_{ij} - (\ln S_j - \ln S_i))$ to be as small as possible for each pair of instances i, j. In other words we want to find the values for S_1, S_2, ---, S_n that minimize

$$Q = \sum_{i=1}^{n} \sum_{j=1}^{n} [\ln W_{ij} - (\ln S_j - \ln S_i)]^2 , \tag{7}$$

or

$$Q = \sum_{i=1}^{n} \sum_{j=1}^{n} [(\ln W_{ij})^2 - 2 \times \ln W_{ij} \times \ln S_j + 2 \times \ln W_{ij} \times S_i$$

$$+ (\ln S_j)^2 - 2 \times \ln S_j \times \ln S_i + (\ln S_i)^2].$$

In order to minimize Q we take the n partial derivatives with respect to S_j, j = 1,2, ---,n, and set them equal to zero.

Thus,

$$\frac{\partial Q}{\partial S_j} = \sum_{i=1}^{n} \sum_{j=1}^{n} \left[- \frac{2 \times \ln W_{ij}}{S_j} + \frac{2 \times \ln S_j}{S_j} - \frac{2 \times \ln S_i}{S_j} \right] = 0,$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} [- \ln W_{ij} + \ln S_j - \ln S_i] = 0, \text{ and}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \ln S_j = \sum_{i=1}^{n} \sum_{j=1}^{n} \ln W_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{n} \ln S_i,$$

which finally gives a new set of equations,

$$\ln S_j = \frac{\sum_{i=1}^{n} \ln W_{ij}}{n} + \frac{\sum_{i=1}^{n} \ln S_i}{n} \; ; \; j=1,2,\cdots,n. \qquad (8)$$

In order to give a solution entirely in terms of the observed W_{ij} it is necessary to specify a unit for the scale value. There will be no loss in generality if the average of the natural logs of the scale values are set at zero, or

$$\frac{\sum_{i=1}^{n} \ln Si}{n} = 0.$$

This gives a simple algebraic expression for the least-squares estimates of the scale values, namely,

$$\ln Sj = \frac{\sum_{i=1}^{n} \ln W_{ij}}{n} \; ; \; j = 1,2,---,n, \tag{9}$$

or alternatively by taking the antilogarithms,

$$S_j = \left[\prod_{i=1}^{n} w_{ij} \right]^{1/n} \; ; \; j = 1,2,---,n. \tag{10}$$

The scale value of instance j, S_j (overall system effectiveness of weapon system j), as derived from the least squares method is simply the geometric mean of the j^{th} column of the W matrix.

The Constant Sum Scaling Method has now formally been established. Applied on the judged data it gave $\overline{A}$ and W matrices for each group (Appendix B).

The values for the judged overall system effectiveness, as shown in Table 2, were obtained from Equation (10).

Table 2
Overall System Effectiveness

Weapon System	Group 1	Group 2	Group 3	Group 4
A	1.906	2.025	1.707	1.892
B	0.559	0.612	0.559	0.577
C	1.435	1.442	1.490	1.452
D	0.939	0.887	0.977	0.931
E	0.510	0.502	0.525	0.510
F	1.243	1.115	1.212	1.188
G	1.102	1.126	1.137	1.120

Table 3 gives a rank order of the judged overall system effectiveness within each group.

Table 3

Rank Order of Overall System Effectiveness

Group 1	Group 2	Group 3	Group 4
S_A	S_A	S_A	S_A
S_c	S_c	S_c	S_c
S_F	S_G	S_F	S_F
S_G	S_F	S_G	S_G
S_D	S_D	S_D	S_D
S_B	S_B	S_B	S_B
S_E	S_E	S_E	S_E

All four groups of judges rank the different SAM weapon systems overall system effectiveness in the same order, with exception of S_{G2} and S_{F2} that changed places. The values of S_g and S_f do not however differ significantly for any of the groups (differences between 0.011 and 0.141), which probably makes it difficult to conclude that System F is substantially different from System G in overall effectiveness.

It should be noted that the top expert group (Group 3) gave the highest ranked system (System A) its lowest score among the groups and the lowest ranked system (System E) its highest score among the groups. In other words it seems like the most experienced judges were the ones to be most careful to draw distinctive conclusions. Figure 3 gives a graphical picture of the results summarized in Table 2.

Having established judged overall system effectiveness values (JOSE), the next step is then to find a functional relationship between the JOSE and the system characteristics, a functional overall system effectiveness (FOSE). This will be the content of the next chapter.

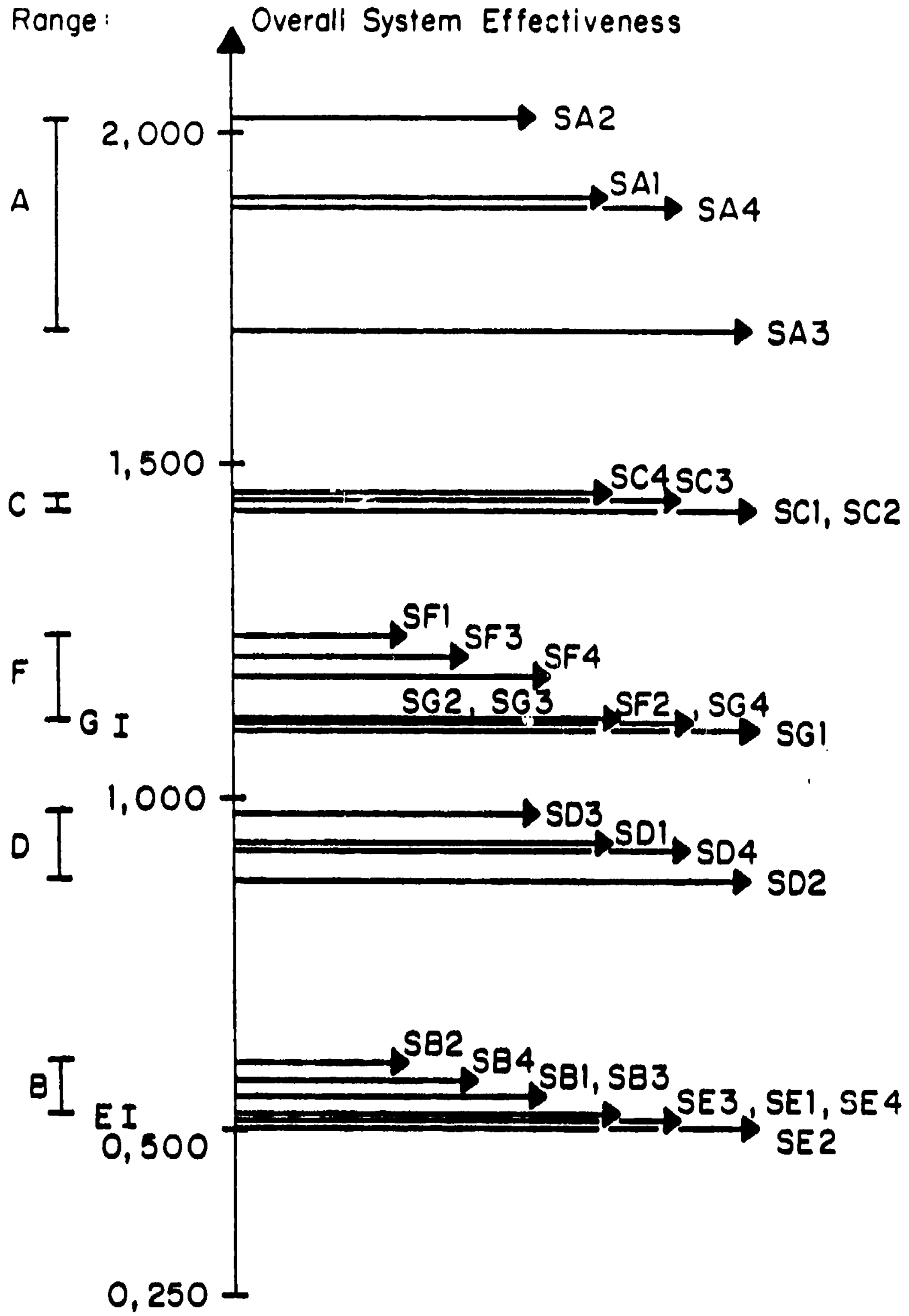


Fig. 3: Graphical representation of the overall grouped system effectiveness.

V. FUNCTIONAL RELATIONSHIP

In the previous chapters overall MOE's for the seven SAM systems were determined within each group of judges. In this chapter a functional relationship between overall-, grouped system effectiveness and system characteristics, as seen in Table 4, will be sought using linear and non-linear multiple regression.

A. FUNCTIONAL RELATIONSHIP BETWEEN OVERALL SYSTEM EFFECTIVENESS AND SYSTEM CHARACTERISTICS

An APL computer program named "REGRESS" taken from OA3660 APL workspace, Public Library Number 2 at the Naval Postgraduate School [6: p. 103] will be used throughout the functional analysis. "REGRESS" does a multiple regression analysis, relating the dependent variable S for overall system effectiveness to the independent variables X_1 to X_5 for system characteristics. The outputs, as seen in Appendix C, give ANOVA tables, coefficients of determination R^2, standard errors SE, regression coefficients (the constant term a and coefficients b_1 to b_5), t - statistics for each coefficient, estimated values for the overall system effectiveness $\hat{S}$, and residuals. In addition plots of residuals versus estimated overall system effectiveness are obtained to see if a particular pattern exists.

Table 4

Overall - Grouped System Effectiveness and System Characteristics

System	Overall System Effectiveness				Characteristics				
	Group 1	Group 2	Group 3	Group 4	Kill Probability	Reaction Time	Max. eff. Range	Average Speed	Price
	S_{i1}	S_{i2}	S_{i3}	S_{i4}	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}
A	1.906	2.025	1.707	1.892	0.90	6	9	2.3	60
B	0.559	0.612	0.559	0.577	0.75	30	12	2.0	60
C	1.435	1.442	1.490	1.452	0.85	10	15	2.2	70
D	0.939	0.887	0.977	0.931	0.70	8	8	2.0	45
E	0.510	0.502	0.525	0.510	0.65	30	22	1.7	80
F	1.243	1.115	1.212	1.188	0.80	12	18	1.5	65
G	1.102	1.126	1.137	1.120	0.80	15	26	1.9	100

Tables 5 through 8 show summaries of the analysis for each group of judges. A part of the analysis was to see if the rank order of the SAM weapon systems obtained by the Constant Sum Scaling Method (Table 3), changed substantially under the functional analysis. Column eight in Tables 5 through 8 summarizes this aspect.

1. Reflections Behind the Choice of Candidate Models

In the process of trying to obtain a transformation of the independent variables that will give a good estimate of a known value, trial and fail may be the most important part. By looking at the data some reflections can however be done, as:

- should all the independent variables have the same impact?
- do some have a positive influence, and others a negative one?
- does any independent variable take a dominant role in form of being significantly more variable than others?
- does any independent variable take a less important role because of little variability?

Such reflections can make it easier to find the right transformation. For this study, the first seven transformation are to be considered more or less as trial and fail (the best among many have been listed). More consideration is however shown for the last six transformations.

Table 5
Summary Of Group 1 Candidate Models

Ref. Page #	REGRESSION MODEL	$R^2 = \frac{RSS}{TSS}$	SE	F-ratio	T-statistics	Residual degrees of freedom	Did the rank order change significantly?
73	$S_{i1} = a + \sum_{j=1}^{5} b_j \times X_{ij}$	0.90408	0.2930	3.1763	very low	1	no
77	$S_{i1} = a + \sum_{j=1}^{4} b_j \times X_{ij} + b_5 \times X_{i5}{}^{1/5}$	0.9842	0.1513	12.468	low	1	no
81	$S_{i1} = a + \ln[\sum_{j=1}^{4} b_j \times X_{ij} + B_5 \times X_{i5}{}^{1/5}]$	0.9667	0.2198	5.8011	very low	1	no
85	$S_{i1} = \exp\,[a + \ln[\sum_{j=1}^{4} b_j \times X_{ij} + b_5 \times X_{i5}{}^{1/5}]]$	0.9979*	0.0583	93.742	very low	1	no
89	$S_{i1} = a + [\sum_{j=1}^{4} b_j \times X_{ij} + b_5 \times X_{i5}{}^{1/5}]^{-1}$	0.9907	0.1161	21.282	very low	1	no
93	$S_{i1} = [a + \sum_{j=1}^{4} b_j \times X_{ij} + b_5 \times X_{i5}{}^{1/5}]^{-1}$	1.0000*	0.0032	7508.9	high	1	no
97	$S_{i1} = [a + \sum_{j=1}^{4} b_j \times X_{ij} + b_5 \times X_{i5}{}^{1/5}]^{-2}$	0.9990*	0.0480	199.62	very low	1	no
101	$S_{i1} = a + b_1 \times (2 \times X_{i1} \times X_{i4}) + b_2 \times (\frac{4}{X_{i2}{}^{1/2}}) + b_3 + (\frac{X_{i5}}{X_{i3}})^{1/2}$	0.9335	0.1792	14.035	low	3	yes
105	$S_{i1} = a + b_1 \times (2 \times X_{i1})^2 \times X_{i4} + b_2 \times (\frac{4}{X_{i2}{}^{1/2}}) + b_3 \times (\frac{X_{i3}}{X_{i5}})^{1/2}$	0.9765	0.1065	41.616	good	3	no
109	$S_{i1} = a + b_1 \times ((2 \times X_{i1})^2 \times X_{i4})^2 + b_2 \times (\frac{4}{X_{i2}{}^{1/2}}) \times (\frac{X_{i3}}{X_{i5}})^{1/2}$	0.9663	0.1105	57.328	good	4	no
113	$S_{i1} = a + b_1 \times ((2 \times X_{i1})^2 \times X_{i4})^4 + b_2 \times (\frac{4}{X_{i2}{}^{1/2}}) \times (\frac{X_{i3}}{X_{i5}})^{1/2}$	0.946	0.1133	54.427	good	4	yes
117	$S_{i1} = a + b_1 \times [4 \times X_{i1} \times (\frac{1}{X_{i2}{}^{1/2}}) \times (\frac{1}{X_{i5}{}^{1/5}})]^2 + b_2 \times (X_{i3} \times X_{i4})^{1/3}$	0.9816	0.0817	106.54	good	4	no
121	$S_{i1} = a + b_1 \times [4 \times X_{i1} \times (\frac{1}{X_{i2}{}^{1/2}}) \times (\frac{1}{X_{i5}{}^{1/5}})]^{3/2} + b_2 \times (X_{i3} \times X_{i4})^{1/3}$	0.9866	0.0697	146.97	good	4	no

*R2 based on a transformed dependent variable

Table 6
Summary Of Group 2 Candidate Models

Ref. Page #	REGRESSION MODEL	$R^2 = \frac{RSS}{TSS}$	SE	F-ratio	T-statistics	Residual degrees of freedom	Did the rank order change significantly?
74	$S_{12} = a + \sum_{j=1}^{5} b_j \times X_{1j}$	0.9329	0.3292	2.7798	very low	1	no
78	$S_{12} = a + \sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}$	0.9909	0.1211	21.809	low	1	no
82	$S_{12} = a + \ln[\sum_{j=1}^{4} b_j \times X_{1j} + B_5 \times X_{15}^{1/5}]$	0.943	0.2399	5.4091	very low	1	no
86	$S_{12} = \exp\ [a + \ln[\sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}]]$	0.9985*	0.0490	136.29	very low	1	no
90	$S_{12} = a + [\sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}]^{-1}$	0.9798	0.1805	9.7072	very low	1	no
94	$S_{12} = [a + \sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}]^{-1}$	0.9967*	0.1127	59.754	very low	1	no
98	$S_{12} = [a + \sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}]^{-2}$	0.9999*	0.0173	1467.4	very low	1	no
102	$S_{12} = a + b_1 \times (2 \times X_{11} \times X_{14}) + b_2 \times (\frac{4}{X_{12}^{1/2}}) + b_3 \times (\frac{X_{15}}{X_{13}})^{1/2}$	0.9289	0.1956	13.068	low	3	yes
106	$S_{12} = a + b_1 \times (2 \times X_{11})^2 \times X_{14} + b_2 \times (\frac{4}{X_{12}^{1/2}}) + b_3 \times (\frac{X_{13}}{X_{15}})^{1/2}$	0.9741	0.1180	37.649	acceptable	3	no
110	$S_{12} = a + b_1 \times ((2 \times X_{11})^2 \times X_{14})^2 + b_2 \times (\frac{4}{X_{12}^{1/2}}) \times (\frac{X_{13}}{X_{15}})^{1/2}$	0.9727	0.1049	71.325	good	4	no
114	$S_{12} = a + b_1 \times ((2 \times X_{11})^2 \times X_{14})^4 + b_2 \times (\frac{4}{X_{12}^{1/2}}) \times (\frac{X_{13}}{X_{15}})^{1/2}$	0.9712	0.1078	67.412	good	4	yes
118	$S_{12} = a + b_1 \times [4 \times X_{11} \times (\frac{1}{X_{12}^{1/2}}) \times (\frac{1}{X_{15}^{1/5}})]^2 + b_2 \times (X_{13} \times X_{14})^{1/3}$	0.9908	0.0609	215.48	good	4	no
122	$S_{12} = a + b_1 \times [4 \times X_{11} \times (\frac{1}{X_{12}^{1/2}}) \times (\frac{1}{X_{15}^{1/5}})]^{3/2} + b_2 \times (X_{13} \times X_{14})^{1/3}$	0.9827	0.0835	113.73	good	4	no

* R2 based on a transformed dependent variable

Table 7
Summary Of Group 3 Candidate Models

Ref. Page #	REGRESSION MODEL	$R^2 = \frac{RSS}{TSS}$	SE	F-ratio	T-statistics	Residual degrees of freedom	Did the rank order change significantly?
75	$S_{13} = a + \sum_{j=1}^{5} b_j \times X_{1j}$	0.9731	0.1774	7.2419	very low	1	no
79	$S_{13} = a + \sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}$	0.9768	0.1649	8.4198	very low	1	no
83	$S_{13} = a + \ln[\sum_{j=1}^{4} b_j \times X_{1j} + B_5 \times X_{15}^{1/5}]$	0.9820	0.1451	10.929	very low	1	no
87	$S_{13} = \exp[a + \ln[\sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}]]$	0.9999*	0.0141	1438.2	good	1	no
91	$S_{13} = a + [\sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}]^{-1}$	1.0000	0.0064	5780.0	high	1	no
95	$S_{13} = [a + \sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}]^{-1}$	1.0000*	0.0141	4389.6	high	1	no
99	$S_{13} = [a + \sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}]^{-2}$	0.9982*	0.0656	109.29	very low	1	no
103	$S_{13} = a + b_1 \times (2 \times X_{11} \times X_{14}) + b_2 \times (\frac{4}{X_{12}^{1/2}}) + b_3 \times (\frac{X_{15}}{X_{13}})^{1/2}$	0.9788	0.0911	46.064	good	3	no
107	$S_{13} = a + b_1 \times (2 \times X_{11})^2 \times X_{14} + b_2 \times (\frac{4}{X_{12}^{1/2}}) + b_3 \times (\frac{X_{13}}{X_{15}})^{1/2}$	0.9955	0.0421	219.06	high	3	no
111	$S_{13} = a + b_1 \times ((2 \times X_{11})^2 \times X_{14})^2 + b_2 \times (\frac{4}{X_{12}^{1/2}}) \times (\frac{X_{13}}{X_{15}})^{1/2}$	0.9704	0.0931	65.517	good	4	yes
115	$S_{13} = a + b_1 \times ((2 \times X_{11})^2 \times X_{14})^4 + b_2 \times (\frac{4}{X_{12}^{1/2}}) \times (\frac{X_{13}}{X_{15}})^{1/2}$	0.9481	0.1233	36.503	good	4	yes
119	$S_{13} = a + b_1 \times [4 \times X_{11} \times (\frac{1}{X_{12}^{1/2}}) \times (\frac{1}{X_{15}^{1/5}})]^2 + b_2 \times (X_{13} \times X_{14})^{1/3}$	0.9547	0.1152	42.128	good	4	no
123	$S_{13} = a + b_1 \times [4 \times X_{11} \times (\frac{1}{X_{12}^{1/2}}) \times (\frac{1}{X_{15}^{1/5}})]^{3/2} + b_2 \times (X_{13} \times X_{14})^{1/3}$	0.9763	0.0833	82.469	good	4	no

* R2 based on a transformed dependent variable

Table 8
Summary Of Group 4 Candidate Models

Ref. Page #	REGRESSION MODEL	$R^2 = \frac{RSS}{TSS}$	SE	F-ratio	T-statistics	Residual degrees of freedom	Did the rank order change significantly?
76	$S_{14} = a + \sum_{j=1}^{5} b_j \times X_{1j}$	0.9471	0.2730	3.5826	very low	1	no
80	$S_{14} = a + \sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}$	0.9854	0.1437	13.454	low	1	no
84	$S_{14} = a + \ln[\sum_{j=1}^{4} b_j \times X_{1j} + B_5 \times X_{15}^{1/5}]$	0.9701	0.2053	6.4884	very low	1	no
88	$S_{14} = \exp[a + \ln[\sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}]]$	0.9989*	0.0412	178.86	very low	1	no
92	$S_{14} = a + [\sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}]^{-1}$	0.9923	0.1040	25.866	very low	1	no
96	$S_{14} = [a + \sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}]^{-1}$	0.9995*	0.0436	402.50	low	1	no
100	$S_{14} = [a + \sum_{j=1}^{4} b_j \times X_{1j} + b_5 \times X_{15}^{1/5}]^{-2}$	0.9997*	0.0265	62.498	very low	1	no
104	$S_{14} = a + b_1 \times (2 \times X_{11} \times X_{14}) + b_2 \times (\frac{4}{X_{12}^{1/2}}) + b_3 \times (\frac{X_{15}}{X_{13}})^{1/2}$	0.9475	0.1570	18.050	T_4 low	3	yes
108	$S_{14} = a + b_1 \times (2 \times X_{11})^2 \times X_{14} + b_2 \times (\frac{4}{X_{12}^{1/2}}) + b_3 \times (\frac{X_{13}}{X_{15}})^{1/2}$	0.9858	0.0816	69.615	good	3	no
112	$S_{14} = a + b_1 \times ((2 \times X_{11})^2 \times X_{14})^2 + b_2 \times (\frac{4}{X_{12}^{1/2}}) \times (\frac{X_{13}}{X_{15}})^{1/2}$	0.9746	0.0946	76.817	good	4	no
116	$S_{14} = a + b_1 \times ((2 \times X_{11})^2 \times X_{14})^4 + b_2 \times (\frac{4}{X_{12}^{1/2}}) \times (\frac{X_{13}}{X_{15}})^{1/2}$	0.9665	0.1086	57.709	good	4	yes
120	$S_{14} = a + b_1 \times [4 \times X_{11} \times (\frac{1}{X_{12}^{1/2}}) \times (\frac{1}{X_{15}^{1/5}})]^2 + b_2 \times (X_{13} \times X_{14})^{1/3}$	0.9889	0.0627	177.48	good	4	no
124	$S_{14} = a + b_1 \times [4 \times X_{11} \times (\frac{1}{X_{12}^{1/2}}) \times (\frac{1}{X_{15}^{1/5}})]^{3/2} + b_2 \times (X_{13} \times X_{14})^{1/3}$	0.9933	0.0487	294.74	good	4	no

* R2 based on a transformed dependent variable

((1)) The candidate model

$$Si = a + \sum_{j=1}^{5} b_j \times X_{ij}, \quad i = 1,2,\text{---},7,$$

is a linear combination of the characteristics.

((2)) The candidate model

$$Si = a + \sum_{j=1}^{4} b_j \times X_{ij} + b_5 \times X_{i5}^{1/5}; \quad i=1,2,\text{---},7,$$

transforms X_5, being the cost of a missile, by using the fifth root (which gave the best result of all applied transformations on X_5).

Any transformation where a linear combination of the independent variables was raised to a power greater than 1.0 gave a bad data fit with unacceptably high standard errors.

Negative powers and logarithmic transformations however gave an overall more satisfying result as shown in Tables 5 through 8.

((3)) The candidate model

$$S_i = a + \ln \left[\sum_{j=1}^{4} b_j \times X_{ij} + b_5 \times X_{i5}^{1/5} \right]; \quad i=1,2,\text{---},7,$$

is the natural log of the linear combination of the characteristics.

((4)) The candidate model

$$S_i = \exp[a + \ln[\sum_{j=1}^{4} b_j \times X_{ij} + b_5 \times X_{i5}^{1/5}]]; \; i=1,2,---,7,$$

is the natural log of both the overall system effectiveness and of the linear combination of the characteristics.

((5)) The candidate model

$$S_i = a + [\sum_{j=1}^{4} b_j \times X_{ij} + b_5 \times X_{i5}^{1/5}]^{-1}; \; i=1,2,---,7,$$

is the reciprocal of the linear combination of the characteristics.

((6)) The candidate model

$$S_i = [a + \sum_{j=1}^{4} b_j \times X_{ij} + b_5 \times X_{i5}^{1/5}]^{-1}; \; i=1,2,---,7,$$

is a linear combination of the characteristics and the reciprocal of the overall system effeciveness.

((7)) The candidate model

$$S_i = [a + \sum_{j=1}^{4} b_j \times X_{ij} + b_5 \times X_{i5}^{1/5}]^{-2}; \; i=1,2,---,7,$$

is a linear combination of the characteristics and a reciprocal transformation of the overall system effectiveness to the second power.

It should be noted that the seven first candidate models have only one residual degree of freedom. Obtained transformations are therefore not very robust and highly sensitive to small changes in the independent variables. Nonlinear combinations of the independent variables will increase the residual degrees of freedom and thus give more robust transformations.

((8)) The candidate model

$$S_i = a + b_1 \times (2 \times X_{i1} \times X_{i4}) + b_2 \times \left(\frac{4}{X_{i2}^{1/2}}\right) + b_3 \times \left(\frac{X_{i5}}{X_{i3}}\right)^{1/2} \quad ; \; i = 1,2,\text{---},7,$$

is a transformation that combines the independent variables X_1 and X_4 in such a manner that the higher the product $(X_1 \times X_4)$, the better the SAM system. The reciprocal of X_2 was used because it was considered that the overall system effectiveness would possess diminishing marginal returns with respect to increasing reaction time, X_2. The 4 in the numerator was chosen to give approximately the same impact from this new second independent variable as for the first new one. As seen in Table 9, X_3 and X_5 are correlated independent variables.

Table 9

Correlation Between Independent Variables

	X_1	X_2	X_3	X_4	X_5
X_1	1.00	0.64	0.20	0.54	0.03
X_2	0.64	1.00	0.37	0.38	0.27
X_3	0.20	0.37	1.00	0.56	<u>0.93</u>
X_4	0.54	0.38	0.56	1.00	0.24
X_5	0.03	0.27	<u>0.93</u>	0.24	1.00

It was therefore concluded that these two variables should be combined. $\left(\frac{X_5}{X_3}\right)^{1/2}$ gives about the same impact as for the other two new independent variables. With three independent variables the residual degrees of freedom increases to three which means a more robust transformation than the former ones.

((9)) The candidate model

$$S_i = a + b_1 x (2 \times X_{i1})^2 \times X_{i4} + b_2 X\left(\frac{4}{X_{i2}^{1/2}}\right) + b_3 x \left(\frac{X_{i3}}{X_{i5}}\right)^{1/2};$$

$$i = 1, 2, ---, 7,$$

shows the same nonlinear combination as ((8)) except that the reciprocal of X_5 is used because of the diminishing marginal returns in overall system effectiveness with respect to increasing cost.

The last four candidate models use nonlinear combinations such that only two new independent variables are applied for the regression analysis. This increases the robustness even further. It should be noted that reciprocals are used both for X_2 and for X_5, using the assumption of diminishing marginal returns with respect to increasing characteristic values for these two variables.

((10)) The candidate model

$$S_i = a+b_1 x((2 \times X_{i1})^2 \times X_{i4})^2 + b_2 \times \left(\frac{4}{X_{i2}^{1/2}}\right) x \left(\frac{X_{i3}}{X_{i5}}\right)^{1/2};$$

$$i=1,2,---,7,$$

is a nonlinear combination of the original independent variables that is constructed by applying obtained knowledge from previous transformations.

((11)) The candidate model

$$S_i = a+b_1 x((2 \times X_{i1})^2 \times X_{i4})^4 + b_2 \times \left(\frac{4}{X_{i2}^{1/2}}\right) x \left(\frac{X_{i3}}{X_{i5}}\right)^{1/2};$$

$$i=1,2,---,7,$$

modifies ((10)) with increased impact on the first new independent variable.

((12)) The candidate model

$$S_i = a+b_1x[4xX_{i1}x(\frac{1}{X_{i2}})^{1/4} \text{ x } (\frac{1}{X_{i5}})^{1/5}]^2 + b_2 \text{ x } (X_{i3} \text{ x } X_{i4})^{1/3} ;$$

$i=1,2,---,7,$

uses the assumption that the higher average missile speed X_4, the longer the maximum effective range X_3, and vice versa. ((13)) The candidate model ((13)) is the same as ((12)), but with 3/2 as exponent of the first new independent variable instead of 2.

Tables 5 through 8, containing all candidate models for each of the four groups of judges, are meant to be a guide for decision makers to select the best equation (transformation) among the presented thirteen. General rules can be applied to assist in the choice.

The coefficient of determination is

$$R^2 = \frac{\text{Regression sum of squares}}{\text{Regression sum of squares + Residual sum of squares}} . \quad (11)$$

The smaller the residual sum of squares (RSS) the better is the candidate model and thus the closer R^2 is to the value 1.0000 (which is considered to be ideal) the better.

The standard error, SE is defined as,

$$SE = (\sum_{i=1}^{n} (Si - \hat{S}i)^2)^{1/2}; \; Si = \text{JOSE}, \; \hat{S}i = \text{FOSE}. \quad (12)$$

The smaller the standard error, the better the candidate model. In Appendix C, standard error can be read for each SAM weapon system, within each group, and for each of the thirteen candidate models.

The F-ratio is defined as

$$F = \frac{\text{Regression mean squares}}{\text{Residual mean squares}} , \qquad (13)$$

and the lower the Residual mean square (RMS) the better is the equation. In other words, the higher the F-ratio the better.

The t-statistics are obtained for the constant a and for each of the regression coefficients b_1 to b_5. Our t-statistic is acceptable if $t_i = \left| \frac{\hat{B}_i - b_i}{\sqrt{V_{ii}}} \right| > t_{1-\alpha}(n-k)$; $i =$ 1,2,---,6, where

$\hat{b}_i$ = estimated i^{th} coefficient,

b_i = i^{th} coefficient given by the null-hypothesis,

V_{ii} = i^{th} diagonal element of the variance - covariance matrix,

$t_{1-\alpha}(n - k)$ = value from t-table with significance level α and (n-k) degrees of freedom, where n = number of SAM weapon systems, and k = number of independent variables.

For $\alpha = 0.05$ and the worst case, $k = 5$, $t_{i-\alpha}(n-k) = 2.920$. Lower values for t_i can give unpredictable results even if the candidate model gives a very small SE and a R^2 close to 1.0000.

Figure 4 through 7 show plots of standard error SE versus R^2 for the thirteen selected candidate models, over each group of judges. If any decision should be made on the basis of SE and R^2 alone, candidate models ((4)), ((6)), and ((7)) seem to be the best. Common to these three however, is that R^2 is based on a transformed dependent variable (S), and is thus not directly comparable to the rest of the candidate models. What can be seen for candidate models with 3 residual degrees of freedom is that model ((9)) is better than ((8)) for every group, based on SE and R^2 alone. Just as easy is it to establish the fact that for candidate models with 4 residuals degree of freedom, ((13)) is better than ((10)) and ((11)) for every group.

B. SELECTION OF THE BEST EQUATION

To select the best candidate model from Tables 5 through 8, seems to be an easy task. The model within each group, that has the R^2 closest to 1.0000, the smallest SE, the highest F-ratio, the largest t-statistics, the highest number of residual degrees of freedom, and no substantial change in rank order of the overall system effectiveness, should be the obvious choice. Such a candidate model however, did not appear in the set using the available data. The solution will therefore be to compromise such that a model that satisfies all basic requirements (high F-ratio, R^2 close to 1.0000, small SE, t-statistics greater than

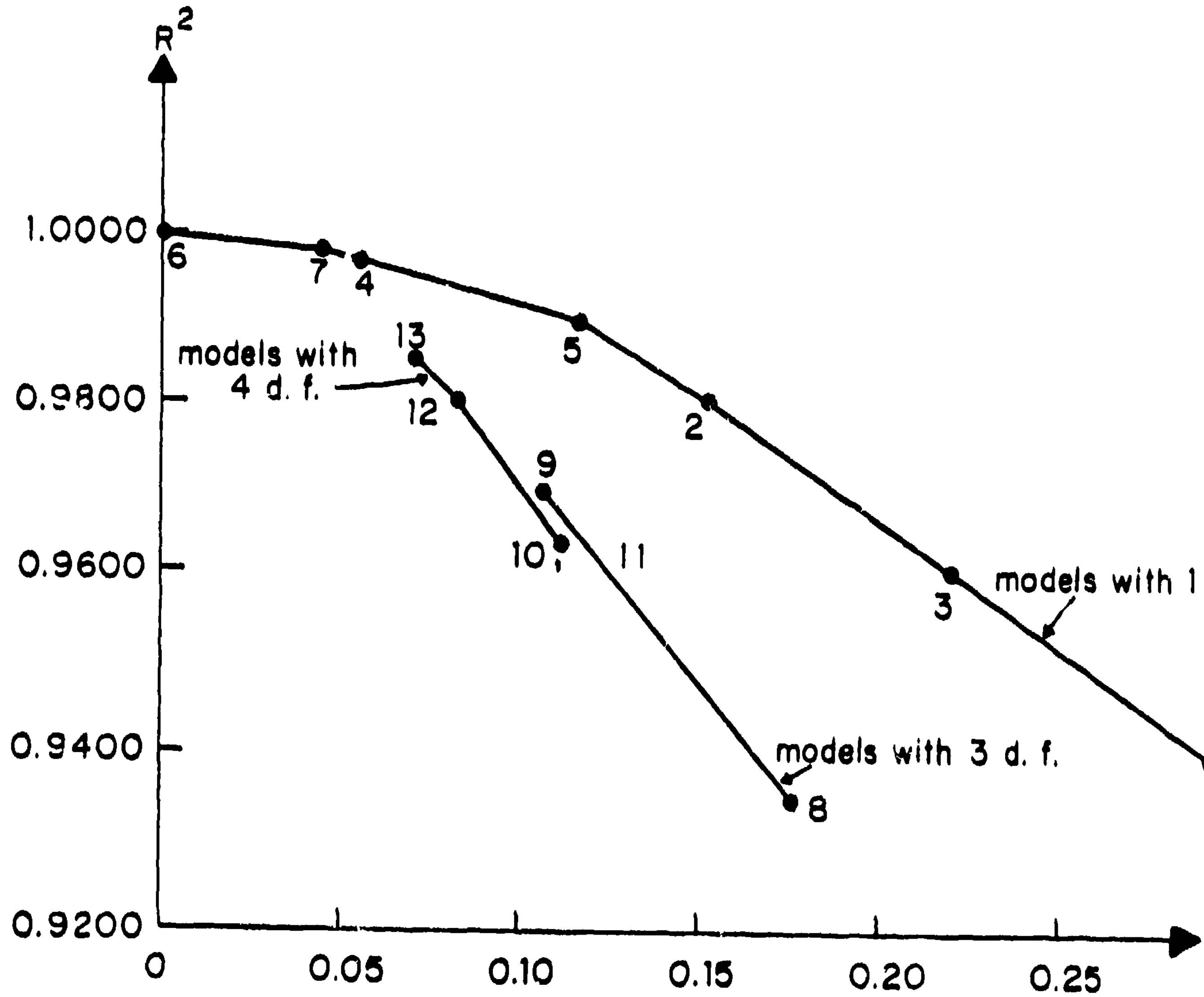


Fig. 4 : R^2 vs. SE for Group 1 Candidate Models.

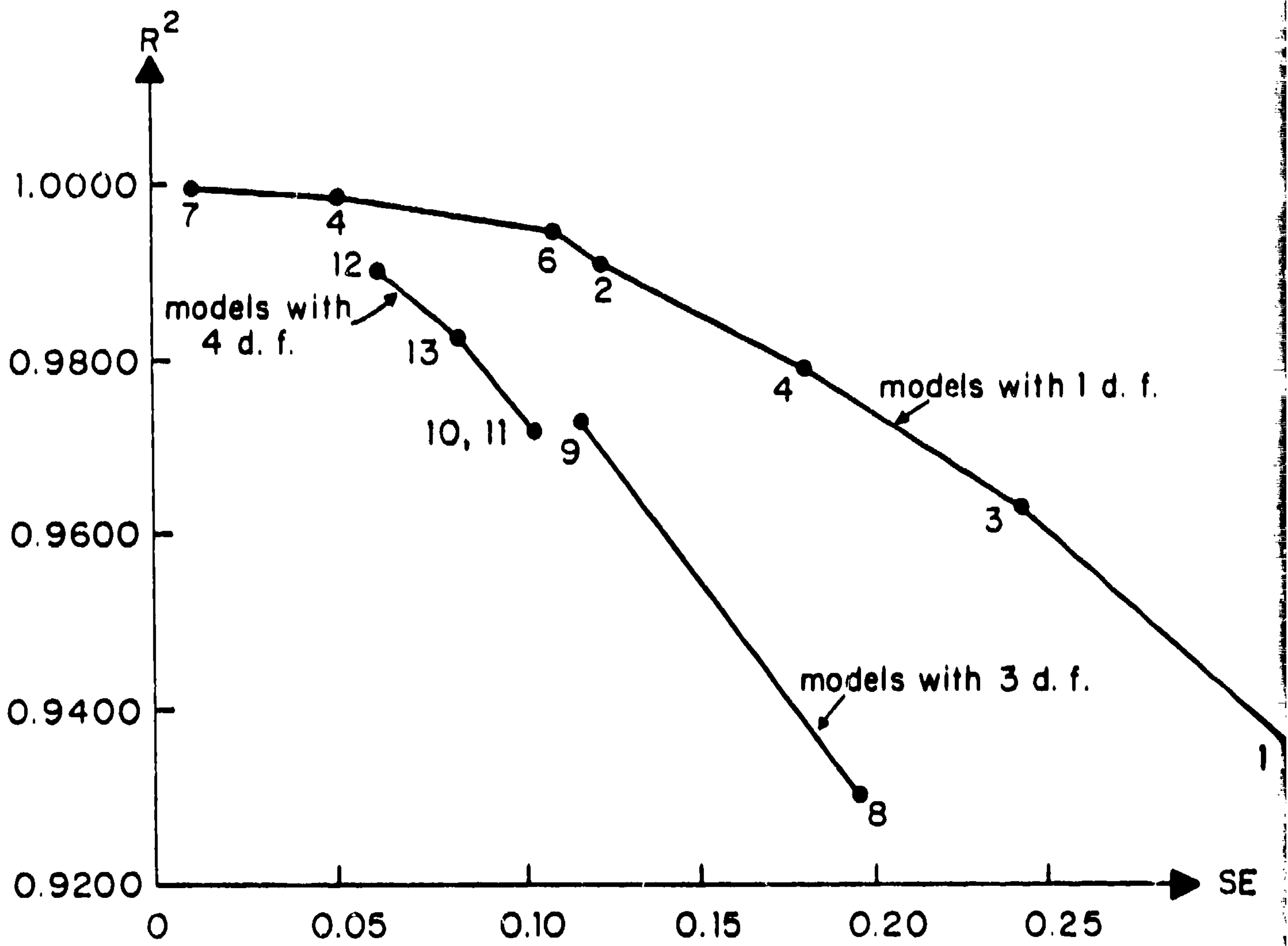


Fig. 5 : R^2 vs SE for Group 2 Candidate Models.

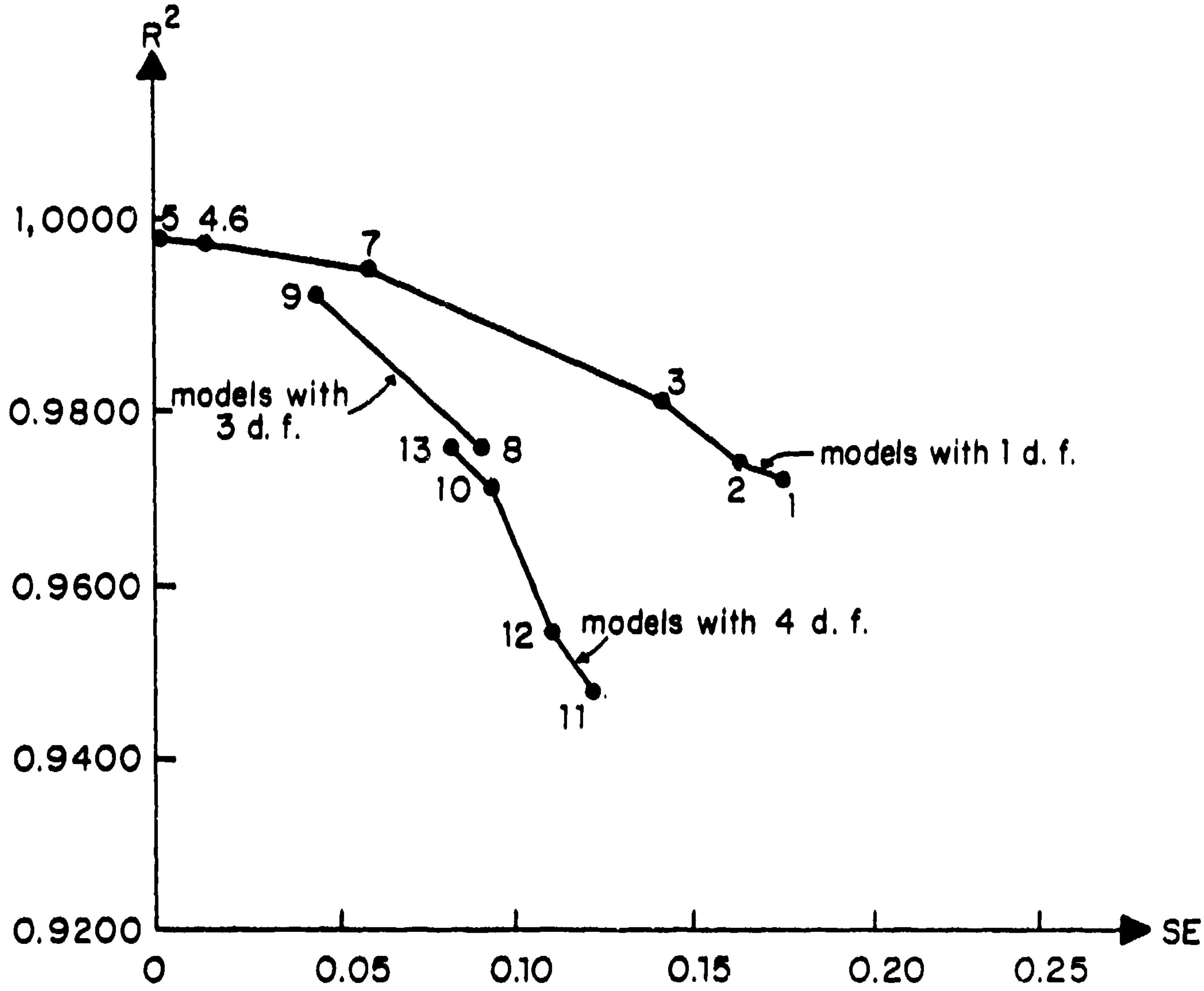


Fig. 6 : R^2 vs. SE for Group 3 Candidate Models.

Table 10
Candidate Models That Satisfy All Basic Requirements
For All Four Groups Of Judges

REGRESSION MODEL	R^2 Max. R^2 Min.	SE Max. SE Min.	F-ratio max. F-ratio min.	T-statistics	Residual degrees of freedom	Did the rank order change significantly within any of the groups?
((9)) $S_i = a + b_1 x (2x\, X_{11})^2 x\, X_{14} + b_2 x\, (\frac{4}{X_{12}^{1/2}}) + b_3\, x\, (\frac{X_{13}}{X_{15}})^{1/2}$	0.9955 0.9741	0.0421 0.1180	219.06 37.65	Acceptable - Good	3	no
((12)) $S_i = a + b_1 x\, 4x\, X_{11} x\, (\frac{1}{X_{12}^{1/2}})\, x\, (\frac{1}{X_{15}^{1/5}})^2 + b_2 x (X_{13} x\, X_{14})^{1/3}$	0.9908 0.3547	0.0609 0.1152	215.48 42.13	Good	4	no
((13)) $S_i = a + b_1 x\, 4\, x\, X_{11} x\, (\frac{1}{X_{12}^{1/2}})\, x\, (\frac{1}{X_{15}^{1/5}})^{3/2} + b_2\, x\, (X_{13}\, x\, X_{14})^{1/3}$	0.9933 0.9763	0.0487 0.0835	294.74 82.47	Good	4	no

THIS

PAGE

IS

MISSING

IN

ORIGINAL

DOCUMENT

48 FIG 7

$\pm$ 2.920) within all four groups, can be chosen as the best. Candidate models ((9)), ((12)), and ((13)) all qualify accordingly - as seen in Table 10. Among the three models, number ((13)) seems to have the in general (over all four groups) R^2 closest to 1.0000, smallest SE, and highest F-ratio. Number ((13)) has also the highest t-statistics of the three, and one more residual degree of freedom more than number ((9)).

Coefficients for the best candidate model, ((13)), are as follows:

Table 11

Coefficients of the Best Candidate Model

Coefficient	Group 1	Group 2	Group 3	Group 4
a	-1.7842	-1.9881	-1.7100	-1.8366
b_1	2.3688	2.5003	2.1366	2.3512
b_2	0.4622	0.5029	0.4811	0.4818

Under the criteria discussed above, the best estimated value for the overall system effectiveness will therefore be obtained by the following functional relationship (using Group 4 as an example):

$$\text{Overall system effectiveness} = -.18366+2.3512 \times [4 \times (\text{kill probability}) \times (\text{reaction time})^{-1/2} \times (\text{missile price})^{-1/5}]^{3/2} + 0.4818 \times [(\text{max effective range}) \times (\text{average missile speed})]^{1/3}.$$

How well the best equztion (model) fits the judged overall effectiveness for each group can be seen from Table 12. With exception of 14.2% deviation for SAM System B by Group 2, all deviations between judged - and functional overall system effectiveness are below 9.0% with a grand average deviation of 3.6%. This suggests that the best equation in general gives a good fit, close to the answers obtained by the Constant Sum Scaling Method.

To improve the result other transformations could be tried. First one might however try to evaluate why the best candidate model did not give an even better prediction than the one achieved. One approach is to check the assumption behind the REGRESS - function. "REGRESS" uses ordinary least squares (OLS) procedure, where $\underset{\sim}{S}=a+\underset{\sim}{X}b + \underset{\sim}{e}$ is the general model, assuming that the residuals ($\underset{\sim}{e}$) are normally distributed with mean O, ($E(e_i) = 0$; $i = 1,2,\cdots,n$) and with variance σ^2, ($\mathrm{Var}(e_i) = \sigma^2$; $i = 1,2,\cdots,n$). To test this assumption "All Possible Subsets Regression" procedure using BMDP9R [7] was applied to model number ((13)). The results are plotted in Appendix D and show that assumptions about normality are not meet entirely for any of the four

Table 12

Judged Overall System Effectiveness (JOSE) vs
Functional Overall System Effectiveness (FOSE)

SAM weapon system	Group 1 JOSE	Group 1 FOSE	Group 1 Percent deviation a)	Group 2 JOSE	Group 2 FOSE	Group 2 Percent deviation a)	Group 3 JOSE	Group 3 FOSE	Group 3 Percent deviation a)	Group 4 JOSE	Group 4 FOSE	Group 4 Percent deviation a)
A	1.906	1.904	0.1%	2.025	1.947	3.9%	1.707	1.793	5.0%	1.892	1.888	0.2%
B	0.559	0.556	0.5%	0.612	0.525	14.2%	0.559	0.586	4.8%	0.577	0.552	4.3%
C	1.435	1.449	1.0%	1.442	1.473	2.1%	1.490	1.412	5.2%	1.452	1.447	0.3%
D	0.939	1.005	7.0%	0.887	0.994	0.8%	0.977	0.967	1.0%	0.931	0.990	6.3%
E	0.510	0.506	0.8%	0.502	0.480	4.4%	0.525	0.571	8.8%	0.510	0.514	0.8%
F	1.243	1.129	1.1%	1.115	1.132	1.5%	1.212	1.110	8.4%	1.188	1.124	5.4%
G	1.102	1.145	3.9%	1.126	1.159	2.9%	1.137	1.168	2.7%	1.120	1.156	3.2%
	Average percent deviation: 2.1%			Average percent deviation: 4.3%			Average percent deviation: 5.1%			Average percent deviation: 2.9%		

Grand average percent deviation: 3.6%

a) Percent deviation $= \frac{(\text{JOSE} - \text{FOSE}) \times 100}{\text{JOSE}}$

groups. This fact does not degrade the accuracy of the estimation obtained by the best equation, neither does it mean that future forecasting will be less accurate. Fisher-statistics can however no longer be used to develop probability results, and F-ratio, confidence intervals, and significance levels cannot be used with the same exactness as if normality was in order.

"All Possible Subsets Regression" also gave an answer to the question: which variables gave most weight to the regression analysis? This aspect is covered in detail in Appendix D.

A functional relationship has now been developed between the overall system effectiveness and the weapon characteristics. The best estimating equation was found by using candidate model number ((13)):

$$Si = a + b_1 \times [4 \times X_{i1} \times (\frac{1}{X_{i2}^{1/2}}) \times (\frac{1}{X_{i5}^{1/5}})]^{3/2}$$

$$+ b_2 \times (X_{i3} \times X_{i4})^{1/3}.$$

In future work with SAM weapon systems (that have the same mission as stated for those used in this paper), this result could assist military decision makers in at least four ways:

- in assessing the impact on overall system effectiveness of modification of one or more weapon characteristics,
- in evaluating the overall system effectiveness of

several systems in a procurement phase,

- in computing overall system effectiveness for existing SAM systems, and
- in evaluating operational criteria for new (unbuilt) systems compared to already existing systems.

In the next and final chapter, the most important results will be summarized, and some recommendations for further studies will be made.

VI. CONCLUSIONS AND RECOMMENDATIONS

The final chapter is meant to be a summary of the "highlights" obtained in the previous chapters, and additionally to give some recommendations for future research.

A. CONCLUSIONS

Finding overall system effectiveness from a multi-criterion environment using seven fictitious SAM weapon systems as an example, was the main purpose of this paper. The Constant Sum Scaling Method was applied to judgment data collected by questionnaires from four groups of judges. Results shows no significant differences in overall system effectiveness ratings from one group to another.

The next step was to build a model which, given the same information the judges had, would accurately reproduce the judged overall system effectiveness. By applying multiple linear and nonlinear regression, thirteen candidate models were examined. These were all evaluated, and a best equation was obtained as follows:

$$S_i = a + b_1 x [4 \times (\text{kill probability}) \times (\text{reaction time})^{-1/2} \times (\text{missile price})^{-1/5}]^{3/2} + b_2 \times [(\text{max effective range}) \times (\text{average missile speed})]^{1/3}$$ where

S_i = overall system effectiveness for weapon system i; i = A, B, ---, G, and where a, b_1, and b_2 are listed in

Table 11 for the four groups. This result of the statistical analysis has a large degree of robustness in it, having four residual degrees of freedom, which makes it less sensitive to changes in weapon characteristics. The grand average percent deviation between judged- and reproduced (functional) overall system effectiveness is 3.6%, which is considered quite acceptable even if the percent deviation in one case is as high as 14.2%.

The main limitation of the obtained results is that only operational weapon characteristics and missile price were selected as independent variables. Other non-operational elements of combat that might be of equal or greater importance are therefore not reflected in the resulting best equation, or in the judged overall system effectiveness.

B. SUGGESTIONS FOR FURTHER WORK

Judgment modeling (Policy Capturing) requires a set of judged overall system effectiveness values associated with a set of independent variables (characteristics) to obtain the implicit weights (functional overall system effectiveness). The applied methodology however, could be taken even further to determine the weights without obtained judgments, called Policy Specifying [2]. This could be done by stating desired properties of the relations among the independent variables in sufficient detail that the numerical weights become known.

If appropriate sensitivity analysis were applied for each of the independent variables the obtained methodology could be used to make decision models for wargaming situations.

An interesting question that has not been answered in this study, is: how would the overall system effectiveness change if one or more of the characteristics were omitted or changed by other characteristics? Another question of interest is: how would existing SAM weapon systems rate compared to the seven fictitious ones used in this study?

Judgment modeling seems to be a procedure that can be efficiently applied to provide additional information for military decision makers. This study has hopefully given a certain feeling for the methodology, and for which applications judgment modeling are useful.

APPENDIX A

QUESTIONNAIRE

(distributed outside the Naval Postgraduate School)

A study is being made of various characteristics of SAM weapon systems, and how they relate to overall operational effectiveness and cost. The objective of the research is to develope a procedure to help military planners:

- evaluate effectiveness of new SAM weapon systems,
- assess the impact of effectiveness by modifying weapon characteristics or changing cost.

The primary operational use of the SAM systems chosen is point to point defense with area defense as a secondary mission.

Essential to the research is information from people with a good theoretical and practical background on SAM weapon systems. In particular, we are interested in subjective rating of overall SAM system effectiveness; these are sought through this questionnaire. The format has been kept short to allow completion in a very short time (five to ten minutes).

If you would like to receive a summary of the results, please fill in the following form.

Name : ______________________________

Address: ______________________________

____

Researcher: K. O. Flaathen, LCDR, Royal Norwegian Navy
Advisor: G. F. Lindsay, Assoc. Prof. of Operations Research, Naval Postgraduate School

QUESTIONNAIRE

(distributed at the Naval Postgraduate School)

A study is being made of various characteristics of Surface to Air Missile weapon systems, and how they relate to overall operational effectiveness and cost. The objective of the research is to develope a procedure to help military planners:

- develop improved methods by which the overall effectiveness of a new weapon system can be assessed,
- assess the impact of effectiveness by modifying weapon characteristics or by changing cost.

The primary operational use of the SAM systems chosen is point to point defense with area defense as secondary mission.

Your participation in this study via completion and return of the enclosed questionnaire before the end of this quarter, will enhance the opportunity for success in my work. Being fully aware of your busy schedule I still hope you will find time to help me. Please return the completed questionnaire to SMC 1403.

If you would like to receive a summary of the results, please fill in the following form.

Name : ____________________

Address: ____________________

Thank you in advance for sharing this portion of your expertise with me.

Knut O. Flaathen
Lieutenant Commander
Royal Norwegian Navy

OVERALL SYSTEM EFFECTIVENESS OF SAM WEAPONS

There are many characteristics (factors) of SAM weapons which serve as measures of effectiveness for such systems. Five important ones are listed in the table below. We have also shown characteristic values for seven fictitious SAM weapons, A - G.

SYSTEM / FACTOR	A	B	C	D	E	F	G
Kill probability of single shot	0.90	0.75	0.85	0.70	0.65	0.80	0.80
Reaction time (seconds from detection to missile launch)	6	30	10	8	30	12	15
Max Effective Range (in km)	9	12	15	8	22	18	26
Average missile speed (in Mach)	2.3	2.0	2.2	2.0	1.7	1.5	1.9
Missile-price (in 10,000 of $)	60	60	70	45	80	65	100

We wish your assessment of the overall system effectiveness of these weapons.

Pairs of the fictitious SAM weapons are listed on the next page. Within each pair, please split 100 points in terms of the relative overall system effectiveness of the two SAM weapon systems.

For example: A __80__ B __20__

if you feel that system A has four times the overall system effectiveness as system B, or A __50__ B __50__ if you feel systems A and B have equally overall effectiveness.

1. A____________________ B____________________

2. A____________________ C____________________

3. A____________________ D____________________

4. A____________________ E____________________

5. A____________________ F____________________

6. A____________________ G____________________

7. B____________________ C____________________

8. B____________________ D____________________

9. B____________________ E____________________

10. B____________________ F____________________

11. B____________________ G____________________

12. C____________________ D____________________

13. C____________________ E____________________

14. C____________________ F____________________

15. C____________________ G____________________

16. D____________________ E____________________

17. D____________________ F____________________

18. D____________________ G____________________

19. E____________________ F____________________

20. E____________________ G____________________

21. F____________________ G____________________

Thank you for your cooperation and prompt return of the completed questionnaire.

APPENDIX B

$\bar{A}$ AND W MATRICES

Table 13: Matrix $\bar{A}$ with elements a_{ijk} denoting the average number of points assigned to weapon system j when compared to weapon system i and judged by Group k.

1. Judged by Group 1:

	A	B	C	D	E	F	G
A	50.00	19.75	44.69	31.33	23.71	40.14	36.65
B	80.25	50.00	70.06	62.50	47.31	68.37	65.71
C	55.31	29.94	50.00	38.22	28.00	45.59	42.78
D	68.67	37.50	61.78	50.00	34.63	55.90	52.04
E	76.29	52.69	72.00	65.37	50.00	71.22	72.43
F	59.86	31.63	54.41	44.10	28.78	50.00	45.45
G	63.35	34.29	57.22	47.96	27.57	54.55	50.00

2. Judged by Group 2:

	A	B	C	D	E	F	G
A	50.00	21.04	40.65	28.20	21.77	37.22	37.73
B	78.96	50.00	67.46	58.60	47.46	62.31	65.22
C	59.35	32.54	50.00	37.75	25.27	43.83	40.69
D	71.80	41.40	62.25	50.00	36.52	54.15	53.48
E	78.23	52.54	74.73	63.48	50.00	70.48	71.75
F	62.78	37.69	56.17	45.85	29.52	50.00	50.22
G	62.27	34.78	59.31	46.52	28.25	49.78	50.00

3. Judged by Group 3:

	A	B	C	D	E	F	G
A	50.00	22.69	46.24	35.46	25.53	40.77	42.67
B	77.31	50.00	72.44	64.81	47.89	68.04	65.18
C	53.76	27.56	50.00	41.05	26.69	42.86	42.10
D	65.54	35.19	58.95	50.00	35.74	54.54	55.45
E	76.47	52.11	73.31	64.26	50.00	72.54	68.32
F	59.23	31.96	57.14	45.46	27.46	50.00	47.49
G	57.33	34.82	57.90	44.55	31.68	52.51	50.00

4. Judged by Group 4:

	A	B	C	D	E	F	G
A	50.00	21.02	43.70	31.38	22.98	39.29	38.69
B	78.98	50.00	69.81	61.77	47.52	66.16	65.39
C	56.30	30.19	50.00	38.84	26.68	44.22	41.86
D	68.62	38.23	61.16	50.00	35.60	54.91	53.49
E	77.02	52.48	73.32	64.40	50.00	71.33	71.05
F	60.71	33.84	55.78	45.09	28.67	50.00	47.69
G	61.31	34.61	58.14	46.51	28.95	52.31	50.00

Table 14: Matrix W with elements w_{ijk} denoting an estimate of the ratio between scale values S_j and S_i when judged by Group k.

1. Judged by Group 1:

	A	B	C	D	E	F	G
A	1.000	0.246	0.808	0.456	0.311	0.671	0.579
B	4.063	1.000	2.340	1.667	0.898	2.162	1.916
C	1.238	0.427	1.000	0.619	0.389	0.838	0.748
D	2.192	0.600	1.616	1.000	0.530	1.268	1.085
E	3.218	1.114	2.571	1.888	1.000	2.475	2.627
F	1.491	0.463	1.193	0.789	0.404	1.000	0.833
G	1.729	0.522	1.338	0.922	0.381	1.200	1.000
$\prod_{A-G} w_{ij1}$	1.467	0.017	12.539	0.646	0.009	4.578	1.970
S_{j1}	1.906	0.557	1.435	0.939	0.510	1.243	1.102

2. Judged by Group 2:

	A	B	C	D	E	F	G
A	1.000	0.266	0.685	0.393	0.278	0.593	0.606
B	3.753	1.000	2.073	1.415	0.903	1.653	1.875
C	1.460	0.482	1.000	0.606	0.338	0.780	0.686
D	2.546	0.706	1.649	1.000	0.575	1.181	1.150
E	3.593	1.107	2.957	1.738	1.000	2.388	2.540
F	1.687	0.605	1.282	0.847	0.419	1.000	1.009
G	1.650	0.533	1.458	0.870	0.394	0.991	1.000
$\prod_{A-G} w_{ij2}$	39.523	0.032	12.942	0.432	0.008	2.137	2.297
S_{j2}	2.025	0.612	1.442	0.887	0.502	1.115	1.126

3. Judged by Group 3:

	A	B	C	D	E	F	G
A	1.000	0.293	0.860	0.549	0.339	0.688	0.744
B	3.407	1.000	2.628	1.842	0.919	2.129	1.872
C	1.163	0.381	1.000	0.696	0.364	0.750	0.727
D	1.820	0.543	1.436	1.000	0.556	1.200	1.245
E	2.995	1.088	2.747	1.798	1.000	2.642	2.157
F	1.453	0.470	1.333	0.834	0.379	1.000	0.904
G	1.344	0.534	1.375	0.803	0.464	1.106	1.000
$\prod_{A-G} w_{ij3}$	42.178	0.017	16.341	0.848	0.011	3.852	2.458
S_{j3}	1.707	0.559	1.490	0.977	0.525	1.212	1.137

4. Judged by Group 4:

	A	B	C	D	E	F	G
A	1.000	0.266	0.776	0.457	0.298	0.647	0.631
B	3.757	1.000	2.312	1.616	0.905	1.955	1.889
C	1.288	0.432	1.000	0.635	0.364	0.792	0.720
D	2.187	0.619	1.575	1.000	0.553	1.218	1.150
E	3.352	1.104	2.748	1.809	1.000	2.488	2.454
F	1.545	0.511	1.261	0.821	0.402	1.000	0.912
G	1.585	0.529	1.389	0.870	0.407	1.100	1.000
$\prod_{A-G} w_{ij4}$	86.870	0.021	13.601	0.606	0.009	3.339	2.209
S_{j4}	1.892	0.577	1.452	0.931	0.510	1.188	1.120

APPENDIX C

MULTIPLE REGRESSION DATA OUTPUTS

Appendix C contains "REGRESSHOW", "REGRESS", "SCAT", "FMT", "STATISTICS", and computer output for each candidate model from all four groups of judges. "REGRESSHOW" is an APL-function that explains the use of the "REGRESS" - function. "SCAT" and "FMT" are other APL- functions necessary as sub-programs for "REGRESS". "STATISTICS, S_i"; $i = 1,2,\text{---},4$, give detailed summary statistics for the judged overall systems effectiveness for all four groups.

REGRESSHOW

SYNTAX: Z←Y REGRESS X

PARAMETER:

ΔINTERCEPT- DETERMINES WHETHER OR NOT AN INTERCEPT TERM IS TO BE INCLUDED. ΔINTERCEPT=1 GIVES AN INTERCEPT TERM, AND ΔINTERCEPT=0 GIVES NO INTERCEPT. (DEFAULT IS 1.)

GROUP: RELATIONS

SUBPROGRAMS: FMT AND SCAT

DESCRIPTION: REGRESS DOES A MULTIPLE REGRESSION ANALYSIS RELATING THE DEPENDENT VARIABLE Y TO A SET OF CARRIERS X. THE LEFT ARGUMENT Y IS A VECTOR OF SIZE N. THE RIGHT ARGUMENT X IS AN N BY K MATRIX CONSISTING OF N OBSERVATIONS ON EACH OF K VARIABLES OR A VECTOR OF SIZE N IF K=1. OUTPUT CONSISTS OF AN ANOVA TABLE, R-SQUARE, STD. ERROR, REGRESSION COEFFICIENTS (THE FIRST COEFFICIENT IS THE CONSTANT TERM IF ΔINTERCEPT=1.), T-STATISTICS, VARIANCE-COVARIANCE MATRIX, DURBIN-WATSON STATISTIC, AND A VECTOR OF PREDICTED Y VALUES AND RESIDUALS. THERE IS AN OPTION THAT ALLOWS THE USER TO INPUT A VECTOR OF X VALUES AND USE THE REGRESSION EQUATION TO FORECAST Y VALUES. THE USER CAN ALSO OBTAIN A SCATTER PLOT OF THE RESIDUALS. WHEN EXECUTION TERMINATES, THE PREDICTED Y VALUES AND THE RESIDUALS RESIDE IN THE N BY 2 MATRIX Z.

```
∇REGRESS[⎕]∇
∇ Z←Y REGRESS X;N;K;C;XPXINV;XPY;BETA;RSS;TSS;S2;ESS;WID;DEP
  X←(2↑(ρX),1)ρX
  X←(0,1-ΔINTERCEPT)↓1,X
  XPXINV←⌹(⍉X)+.×X
  BETA←XPXINV+.×XPY←(⍉X)+.×Y
  RSS←((⍉BETA)+.×XPY)-C←((+/Y)*2)÷N←ρ,Y
  ESS←(TSS←((⍉Y)+.×Y)-C)-RSS
  S2←,ESS÷(N-1)-K←(ρ,BETA)-ΔINTERCEPT
  CR
  '                           ANOVA'
  C̲H̲←'SOURCE,DF,SUM SQUARES,MEAN SQUARE,F-RATIO'
  ' '
  '⎕REGRESSION⎕,I4,BE16.4' FMT(K),(,RSS),(,RSS÷K),(,RSS÷K)÷S2
  C̲H̲←''
  '⎕  RESIDUAL⎕,I4,BE16.4' FMT((N-1)-K),(,ESS),S2,0
  '⎕TOTAL      ⎕,I4,BE16.4' FMT(N-1),(,TSS),0,0
  ' '
  (⍕'R SQUARE:  '),⍕,RSS÷TSS
  (⍕'STD ERROR: '),⍕,S2*0.5
  C̲H̲←'COEFFICIENTS,T STATISTICS'
  'F15.4' FMT⍉(2,ρ,BETA)ρ(,BETA),(,BETA)÷(1 1 ⍉V←S2×XPXINV)*0.5
  'DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?'
  →A1×⍳'Y'≠1↑⍞
  'VARIANCE-COVARIANCE MATRIX: ',C̲H̲←''
  'E12.4' FMT V
A1:(⍕'DURBIN-WATSON: '),⍕(+/((1↓,C)-(¯1↓,C))*2)÷+/(,C←Y-X+.×BETA)*2
  Z←⍉(2,N)ρ(,X+.×BETA),,C
B1:'DO YOU WANT TO FORECAST A VALUE FOR Y?'
  →C1×⍳'Y'≠1↑⍞
  (⍕'ENTER X VECTOR ('),(⍕K),⍕' VALUES)'
  (⍕'FORECAST OF Y VALUE: '),⍕(C←(1-ΔINTERCEPT)↓1,⎕)+.×BETA
  (⍕'VARIANCE OF FORECAST ERROR: '),⍕S2×1+C+.×XPXINV+.×⍉C
  →B1
C1:'DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?'
  →0×⍳'N'=1↑⍞
  DEP←0.5×WID←⌊/70,(⌈/((0.75×N),30))
  SCAT Z
∇
```

```
      ∇SCAT[⎕]∇
      ∇ W←SCAT Z;N;X;Y;C;R;U;S;L;I;J;K;UT;CL;G;D;B;A;O;V
[1]   →3×ι(2≠+/2≠N)∨(×/N)>+/N←ρZ
[2]   Z←⍉(2,ρZ)ρ(ιρZ),Z←,Z
[3]   Y←Z[;1+ιC←¯1+(ρZ)[2]]
[4]   R←ρZ←X←,Z[;1]
[5]   L←U←S←2ρ0
[6]   J←1+0×ρ(D←NDIVX,NDIVY),B←WID,DEP
[7]   UT←10*⌊10⍟CL←1E¯20+((U[J]←⌈/Z)-S[J]←⌊/Z)÷D[J]
[8]   S[J]←UT×⌊S[J]÷UT←UT[1↑⍋|CL-UT←(1 2 5)×UT]
[9]   U[J]←UT×⌈U[J]÷UT
[10]  L[J]←1+G×⌊(B[J]-1)÷G←(U[J]-S[J])÷UT
[11]  Z←,Y
[12]  →7×ι3>J←J+1
[13]  A←(⌽L)ρ0
[14]  X←1+⌊0.5+(L[1]-1)×(X-S[1])÷U[1]-S[1]
[15]  Y←1+⌊0.5+(L[2]-1)×(Y-S[2])÷U[2]-S[2]
[16]  I←1
[17]  →20×ι1<C
[18]  A[Y[I;1];X[I]]←10|A[Y[I;1];X[I]]+1
[19]  →18+6×R<I←I+1
[20]  J←1
[21]  D←0=V←A[Y[I;J];X[I]]
[22]  A[Y[I;J];X[I]]←(10×V>K+1)+((K+1)×K=V)+(K←35-2×J)×D
[23]  →21×ιR≥I←I+1
[24]  →21×ιC≥J←J+I←1
[25]  O←(⌽ρA)⌊1⌈1+⌊0.5+(L-1)×S÷S-U
[26]  A[;O[1]]←A[;O[1]]+36×0=A[;O[1]]
[27]  A[O[2];]←A[O[2];]+35×0=A[O[2];]
[28]  W←' ∘23456789⍒LL̲KK̲JJ̲II̲HH̲GG̲FF̲EE̲DD̲CC̲BB̲AA̲-|'[1+⊖A]
[29]  (⍕'RANGE OF X: '),⍕S[1],U[1]
[30]  (⍕'RANGE OF Y: '),⍕S[2],U[2]
      ∇
```

```
        ∇FMT[⎕]∇
        ∇ OL←E̲ FMT R;S;W;Δ;G;X;T;K;J;M;Q;P;D;N;O;L;B;V;CH;H
[1]      N←Q←1↓M←⍴R←(1⌈¯2↑⍴H)⍴R
[2]      OL←((1 =1↑M)↓ 1 0 ×M←M+2↑H←1<⍴CH←C̲H̲,',')⍴Δ←'0123456789.'
[3]      →E×⍳(N←0 =N)∨V←1≥⍴S←,E̲
[4]     L0:→~⍳V∨(×P←4×Q =⍴K←⍴X←' ')∧∨/('A',O←']') ∊S
[5]      →(L0+(V←0 =⍴S←J↓S)+⍳B =M[2]←1),L̲-(⍳×B←O+.=K),P×~'A' ∊K←K,(J←S⍳',')↑S
[6]      →E+×⍴S←'TEXT DELIMITER'
[7]      →L3-3××(⍴G←K ≠K←(K ∊¯1↓Δ)/K)⌊W←⍴X←(⍴K←(K⍳O)↑K)↓(-(⌽K)⍳O)↓K
[8]     L̲:→(D←~1↑G←K ∊Δ)/L3-2×(⍴K)≠W←1↑O←'XA' ∊K←(~K ∊' ,')/K
[9]      →L3×⍳(B≠+/G)∧×M[2]←10⊥|1-Δ⍳(B←|1-G⍳0)↑K
[10]     →L3-⌽O,-(L←'EFI' ∊K)/×W←10⊥|1-Δ⍳(|1-G←B⍳'.')↑B←(1-(⌽G)⍳0)↑K
[11]     A̲←(1↓⍴X←((1⌈⍴A̲)⌊(M[1]-H),W)↑A̲)⌽A̲
[12]    L3:→(HD×⍳H∧~'X' ∊K),E-⍴X←-W,D←0⍴P←((M-H,0)×1,W)⍴X
[13]     →L4-⍳~1↑L,Q←1↓⍴R←(0 1 ×⍴P←R[;⍳M[2]←Q⌊M[2]⌈Q×V∧D])↓R
[14]     P←P÷10*L←⌊10⍟|P+0 =P
[15]     →L3×⍳0 =J←+/B←('B' ∊K)∧0 =P←(⌊0.5+N̲×,P)÷N←10*D←10⊥|1-Δ⍳G↑B
[16]    L4:→(⍴1↓⍴L)/F-⍴⍴X←(1 0 ×⍴G←J⍴T\'¯')⍴J←J,O←∨/T←0>P←B/P
[17]     →(×L←(O⌈L×J←'Z' ∊K)⌈.×~T←(T+O←1+⌊10⍟1⌈|P)>O+L←W-D+O+~2↑L)/L/F,F,I
[18]     →E+×⍴S←'FIELD WIDTH'
[19]     →L4+1+⍳((J[2]←L∨.<0)+O←1+10⌈.≤|L←(B/,L)+T←10 =|P)>W-D+O+3
[20]     T←~J←P[T/⍳1↑J]←L←⍴1⍴X←'E','+0¯'[J⍴2-×L],Δ[1+⍉(O⍴10)⊤|L]
[21]    F:→(J∨2≥D×~'T' ∊K)/I,N←⍴X←Δ[11,1+⍉(D⍴10)⊤N×1||P],X
[22]     D←,(-N)↑(D⌈.×⍉X[;2+D]≠1↑Δ)∘.<D←⍳D-1
[23]     X←N⍴X,X[D/⍳⍴X←,X]←' '
[24]    I:→(J←J∨0 =+/O←0⌈L-O)/I+⍴D←⍴P←G,Δ[1+⍉(L⍴10)⊤|P]
[25]     P←D⍴(,O←G⌽O)\(,O←O∘.<(-G)⌽⍳L+G←1↓⍴G)/,P
[26]     →HD-⍳J∨L←~'L' ∊K,P[T/⍳D←1↑X←⍴P←P,X;]←'*'
[27]     P←X⍴(,⌽O)\(,O←~X↑~O)/,P
[28]     →(~H)/F-N←1,D←0⍴P←B⍀(D,X←W×1-2×L)↑P
[29]    HD:CH←(⍴K←(¯1↑D←0,(M[2]⌊⍴D)⍴D←(',' =CH)/⍳⍴CH)⍴CH)⍴CH
[30]     D←,(M[2],X)↑ 0 ¯1 ↓(M[2],B)⍴(,⌽D∘.≥⍳E←⌈/D←1↓D-¯1⌽D)\K
[31]     →(L0-V∧×Q),⍴OL←OL,((1 =1↑M)↓M×1,W)⍴D,,P
[32]    E:K←'NO VALID E, I, OR F PHRASE'
[33]     (⍕'FMT PROBLEM      ',K),⍕(1,⍴S)⍴S
        ∇
```

STATISTICS ,S1

MEAN: 1.099142857
VARIANCE: 0.2415018095
STD. DEV.: 0.4914283361
COEFF. OF VARIATION: 0.4471014235
LOWER QUARTILE: 0.559
UPPER QUARTILE: 1.435
MEDIAN: 1.102
TRIMEAN: 1.0495
MIDMEAN: 1.0556
RANGE: 1.396
MIDRANGE: 1.208
MEAN ABSOLUTE DEVIATION: 0.368
INTERQUARTILE RANGE: 0.876
COEFF. OF SKEWNESS: 0.2808085241
COEFF. OF KURTOSIS: ¯1.16188856

STATISTICS ,S2

MEAN: 1.101285714
VARIANCE: 0.2690992381
STD. DEV.: 0.5187477596
COEFF. OF VARIATION: 0.4710383081
LOWER QUARTILE: 0.612
UPPER QUARTILE: 1.442
MEDIAN: 1.115
TRIMEAN: 1.071
MIDMEAN: 1.0364
RANGE: 1.523
MIDRANGE: 1.2635
MEAN ABSOLUTE DEVIATION: 0.3702857143
INTERQUARTILE RANGE: 0.83
COEFF. OF SKEWNESS: 0.5796820095
COEFF. OF KURTOSIS: ¯0.8597402161

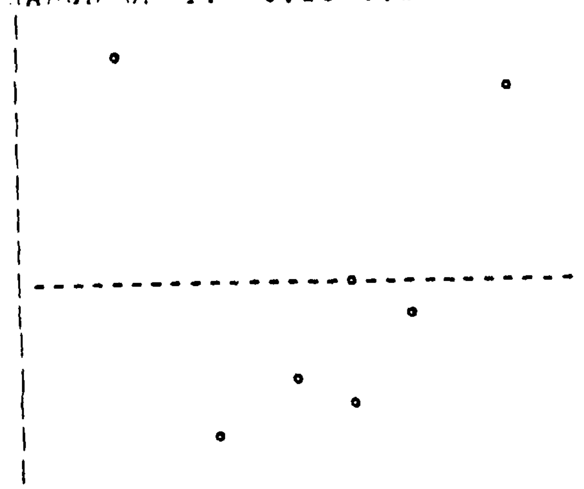
STATISTICS ,S3

MEAN: 1.086714286
VARIANCE: 0.1952769048
STD. DEV.: 0.441901465
COEFF. OF VARIATION: 0.4066399704
LOWER QUARTILE: 0.559
UPPER QUARTILE: 1.49
MEDIAN: 1.137
TRIMEAN: 1.08075
MIDMEAN: 1.075
RANGE: 1.182
MIDRANGE: 1.116
MEAN ABSOLUTE DEVIATION: 0.3354285714
INTERQUARTILE RANGE: 0.931
COEFF. OF SKEWNESS: ¯0.03702800775
COEFF. OF KURTOSIS: ¯1.461561429

STATISTICS ,S4

MEAN: 1.095714286
VARIANCE: 0.2348955714
STD. DEV.: 0.4846602639
COEFF. OF VARIATION: 0.4423235786
LOWER QUARTILE: 0.577
UPPER QUARTILE: 1.452
MEDIAN: 1.12
TRIMEAN: 1.06725
MIDMEAN: 1.0536
RANGE: 1.382
MIDRANGE: 1.201
MEAN ABSOLUTE DEVIATION: 0.3591428571
INTERQUARTILE RANGE: 0.875
COEFF. OF SKEWNESS: 0.3015169357
COEFF. OF KURTOSIS: ¯1.160262769

```
Z←S1 REGRESS X

                         ANOVA

    SOURCE  DF      SUM SQUARES     MEAN SQUARE        F-RATIO
REGRESSION   5       1.3632E00       2.7264E¯1        3.1763E00
  RESIDUAL   1       8.5833E¯2       8.5833E¯2
TOTAL        6        1.4490E0

R SQUARE:  0.9407642333
STD ERROR: 0.2929731542
   COEFFICIENTS    T STATISTICS
        ¯0.0414          ¯0.2799
         3.2755           1.2603
        ¯0.0232          ¯1.2953
        ¯0.0253          ¯0.1161
        ¯0.1752          ¯0.039
         0.0087           0.1189
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.238803758
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0 2
RANGE OF Y: ¯0.15 0.2
```

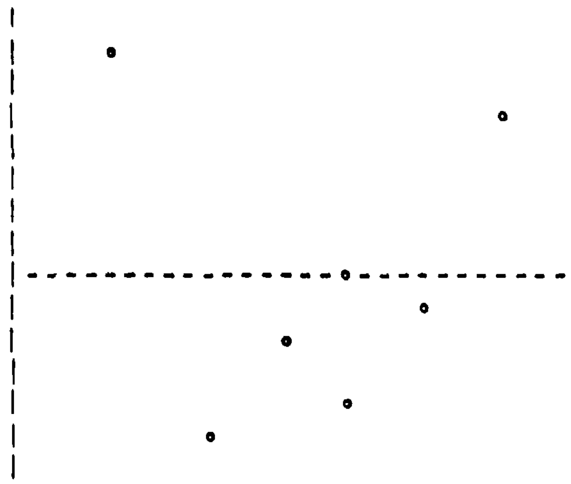


```
     S1,Z
  1.906            1.758734787      0.1472652131
  0.559            0.6863742831    ¯0.1273742831
  1.435            1.454673938     ¯0.01967339381
  0.939            1.004459243     ¯0.06545924257
  0.51             0.3322835812     0.1777164188
  1.243            1.247556831     ¯0.004556831251
  1.102            1.209917337     ¯0.1079173369
```

```
      Z←S2 REGRESS X

                                   ANOVA

    SOURCE  DF      SUM SQUARES       MEAN SQUARE          F-RATIO
REGRESSION   5       1.5062E00         3.0125E¯1         2.7798E00
  RESIDUAL   1       1.0837E¯1         1.0837E¯1
TOTAL        6        1.6146E0

R SQUARE:  0.9328822788
STD ERROR: 0.3291929007
   COEFFICIENTS    T STATISTICS
        ¯1.0915        ¯0.2889
         3.189          1.092
        ¯0.0206        ¯1.0225
        ¯0.0723        ¯0.2951
        ¯0.2851        ¯0.1435
         0.0252         0.3061
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.238803758
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0 2
RANGE OF Y: ¯0.15 0.2
|   •
|                        o
|
|
|
|
|
|
|---------------o-----------
|                   o
|
|            o
|
|                o
|          o
      S2,Z
  2.025          1.859528652    0.1654713477
  0.612          0.7551213376  ¯0.1431213376
  1.442          1.464106192   ¯0.02210619184
  0.887          0.9605518515  ¯0.07355185152
  0.502          0.3023128186   0.1996871814
  1.115          1.120120184   ¯0.00512018414
  1.126          1.247258964   ¯0.1212589641
```

```
Z←S3 REGRESS X

                        ANOVA

   SOURCE    DF       SUM SQUARES        MEAN SQUARE          F-RATIO
REGRESSION    5        1.1402E00          2.2803E¯1          7.2419E00
  RESIDUAL    1        3.1488E¯2          3.1488E¯2
TOTAL         6         1.1717E0

R SQUARE:  0.9731251315
STD ERROR: 0.1774492794
   COEFFICIENTS    T STATISTICS
        ¯1.6304        ¯0.8005
         3.0525         1.9391
        ¯0.0229        ¯2.1083
         0.0596         0.4515
         0.5098         0.4759
        ¯0.0178        ¯0.4024
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.238803758
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 1.8
RANGE OF Y: ¯0.1 0.15
|
|
o                       o
|
|
|
|---------------o-----o------
|
|           o
|    o          o
|

    S3,Z
  1.707          1.617803753      0.08919624744
  0.559          0.6361486207    ¯0.0771486207
  1.49           1.501916198     ¯0.01191619809
  0.977          1.016647644     ¯0.03964764437
  0.525          0.4173599313     0.1076400687
  1.212          1.214760002     ¯0.002760001764
  1.137          1.202363851     ¯0.06536385127
```

```
Z←S4 REGRESS X

                              ANOVA

   SOURCE     DF      SUM SQUARES       MEAN SQUARE          F-RATIO
REGRESSION     5       1.3349E00        2.6697E¯1         3.5826E00
  RESIDUAL     1       7.4519E¯2        7.4519E¯2
TOTAL          6        1.4094E0

R SQUARE:  0.947126355
STD ERROR: 0.2729811537
   COEFFICIENTS    T STATISTICS
        ¯1.1827         ¯0.3775
         3.178           1.3123
        ¯0.0223         ¯1.3314
        ¯0.0186         ¯0.0914
        ¯0.0246         ¯0.015
         0.0072          0.1061
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.238803758
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0 2
RANGE OF Y: ¯0.15 0.2
```



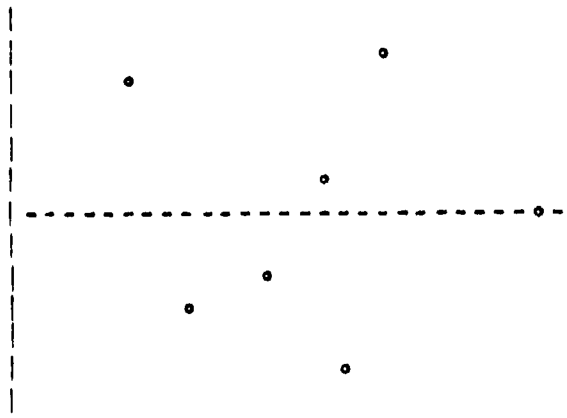
```
      S4,Z
  1.892            1.75478392        0.1372160801
  0.577            0.695682474      ¯0.118682474
  1.452            1.470331421      ¯0.01833142131
  0.931            0.9919924128     ¯0.0609924128
  0.51             0.3444106539      0.1655893461
  1.188            1.192245881      ¯0.004245880671
  1.12             1.220553237      ¯0.1005532375
```

```
      Z←S1 REGRESS X1,X2,X3,X4,X5*÷5

                                   ANOVA

    SOURCE  DF       SUM SQUARES       MEAN SQUARE           F-RATIO
REGRESSION   5        1.4261E00         2.8523E¯1          1.2468E+1
  RESIDUAL   1        2.2877E¯2         2.2877E¯2
TOTAL        6         1.4490E0

R SQUARE:  0.9842121155
STD ERROR: 0.151250838
   COEFFICIENTS   T STATISTICS
       ¯22.1228        ¯1.774
        ¯0.6279        ¯0.2353
        ¯0.0471        ¯2.766
        ¯0.2588        ¯1.6669
        ¯1.713         ¯1.58
        13.7342         1.6748
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.555437999
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0 2
RANGE OF Y: ¯0.1 0.1
|
|                  o
|     o
|
|
|                o
|-------------------------o-
|
|             o
|         o
|
|                o
|
      S1,Z
   1.906          1.908572681      ¯0.002572681461
   0.559          0.6102116769     ¯0.05121167687
   1.435          1.345202512       0.0897974882
   0.939          0.9711426718     ¯0.03214267177
   0.51           0.4432836182      0.06671638184
   1.243          1.232421343       0.01057865662
   1.102          1.183165497      ¯0.08116549679
```

```
      Z←S2 REGRESS X1,X2,X3,X4,X5*÷5

                            ANOVA

    SOURCE  DF      SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION   5       1.5999E00        3.1998E¯1        2.1809E+1
  RESIDUAL   1       1.4672E¯2        1.4672E¯2
TOTAL        6        1.6146E0

R SQUARE:  0.9909129315
STD ERROR: 0.1211277809
   COEFFICIENTS   T STATISTICS
       ¯28.5988        ¯2.8636
        ¯1.4806        ¯0.6927
        ¯0.0496        ¯3.6376
        ¯0.3276        ¯2.6344
        ¯1.9223        ¯2.2139
        17.4721         2.6605
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.555437999
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0 2.5
RANGE OF Y: ¯0.08 0.08
|             o
|   o
|
|
|----------o--------o-----
|        o
|      o
|         o
|
      S2,Z
   2.025           2.027060307    ¯0.002060307243
   0.612           0.6530123796   ¯0.04101237963
   1.442           1.370086544     0.07191345607
   0.887           0.91274115     ¯0.02574115001
   0.502           0.4485708263    0.05342917369
   1.115           1.106528184     0.008471815565
   1.126           1.191000609    ¯0.06500060865
```

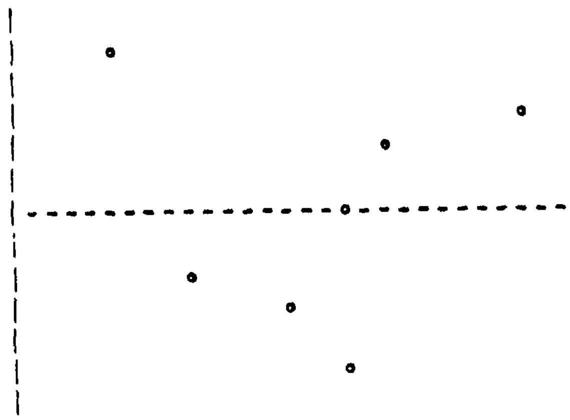
```
Z←S3 REGRESS X1,X2,X3,X4,X5**5

                                 ANOVA

    SOURCE   DF       SUM SQUARES       MEAN SQUARE           F-RATIO
REGRESSION    5        1.1445E00          2.2890E¯1          8.4198E00
  RESIDUAL    1        2.7185E¯2          2.7185E¯2
TOTAL         6         1.1717E0

R SQUARE:  0.9767975572
STD ERROR: 0.1648799783
   COEFFICIENTS    T STATISTICS
         ¯8.896          ¯0.6544
          1.114           0.3829
         ¯0.0341         ¯1.8366
         ¯0.0925         ¯0.5462
         ¯0.5579         ¯0.472
          5.2572          0.5881
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.555437999
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 1.8
RANGE OF Y: ¯0.1 0.1
|                   o
|
|o
|
|
|               o
|-------------------------o--
|
|           o
|   o
|
|                o
|
    S3,Z
  1.707          2.027060307      ¯0.002060307243
  0.559          0.6530123796     ¯0.04101237963
  1.49           1.370086544       0.07191345607
  0.977          0.91274115       ¯0.02574115001
  0.525          0.4485708263      0.05342917369
  1.212          1.106528184       0.008471815565
  1.137          1.191000607      ¯0.06500060865
```

```
Z←S4 REGRESS X1,X2,X3,X4,X5*÷5

                              ANOVA

   SOURCE    DF      SUM SQUARES        MEAN SQUARE          F-RATIO
REGRESSION    5       1.3887E00         2.7775E¯1          1.3454E+1
  RESIDUAL    1       2.0644E¯2         2.0644E¯2
TOTAL         6        1.4094E0

R SQUARE:  0.9853521321
STD ERROR: 0.1436812995
   COEFFICIENTS   T STATISTICS
        ¯20.7098       ¯1.7482
         ¯0.443        ¯0.1747
         ¯0.0443       ¯2.7428
         ¯0.2366       ¯1.6036
         ¯1.4627       ¯1.4201
         12.6821        1.628
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.555437999
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0 2
RANGE OF Y: ¯0.1 0.1
```



```
    S4,Z
 1.892              1.894443928       ¯0.002443928386
 0.577              0.6256487241      ¯0.04864872413
 1.452              1.36669654         0.08530346001
 0.931              0.9615340514      ¯0.03053405136
 0.51               0.4466225234       0.06337747663
 1.188              1.177950766        0.01004923445
 1.12               1.197103467       ¯0.07710346747
```

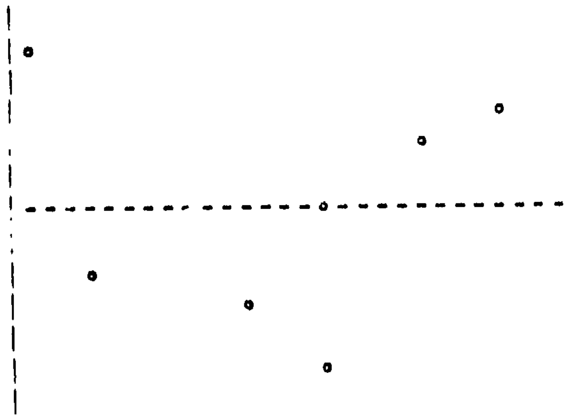
```
      Z←S1 REGRESS ⍉(X1,X2,X3,X4,X5*÷5)

                              ANOVA

     SOURCE  DF      SUM SQUARES       MEAN SQUARE          F-RATIO
 REGRESSION   5       1.4007E00         2.8014E¯1          5.8011E00
   RESIDUAL   1       4.8291E¯2         4.8291E¯2
 TOTAL        6        1.4490E0

 R SQUARE:  0.9666728451
 STD ERROR: 0.2197530645
    COEFFICIENTS    T STATISTICS
          0.2234           0.0498
          2.1264           1.6695
         ¯0.4718          ¯2.0453
         ¯0.5841          ¯0.4264
         ¯0.6856          ¯0.4362
          5.5396           0.5368
 DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
 N
 DURBIN-WATSON: 2.409325477
 DO YOU WANT TO FORECAST A VALUE FOR Y?
 N
 DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
 Y
 RANGE OF X: 0 2
 RANGE OF Y: ¯0.15 0.15
```



```
      S1,Z
  1.906          1.835778593       0.07022140706
  0.559          0.6166087492     ¯0.0576087492
  1.435          1.376130584       0.05886941571
  0.939          1.011555822      ¯0.07255582186
  0.51           0.3884127952      0.1215872048
  1.243          1.23518895        0.007811050498
  1.102          1.230324507      ¯0.128324507
```

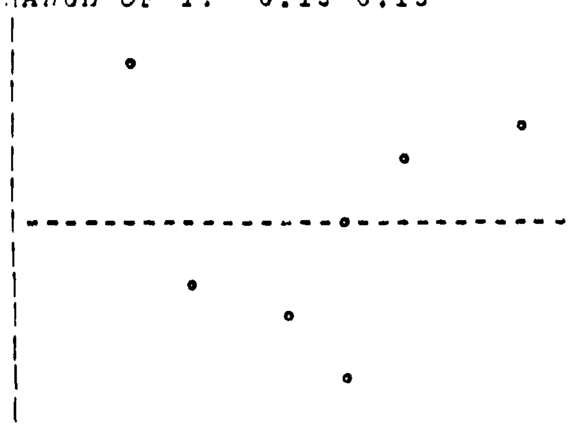
```
Z←S2 REGRESS ●(X1,X2,X3,X4,X5*÷5)

                              ANOVA

   SOURCE  DF      SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION  5        1.5570E00        3.1141E¯1        5.4091E00
  RESIDUAL  1        5.7570E¯2        5.7570E¯2
TOTAL       6         1.6146E0

R SQUARE:  0.9643437624
STD ERROR: 0.2399383216
   COEFFICIENTS   T STATISTICS
        ¯1.9553        ¯0.3991
         2.1487         1.5451
        ¯0.4127        ¯1.6387
        ¯1.1114        ¯0.7431
        ¯0.7317        ¯0.4264
         9.6729         0.8585
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.409325477
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0 2
RANGE OF Y: ¯0.15 0.15
|
|     o
|
|                  o         o
|
|
|
|---------------o----------------
|
|
|          o     o
|
|
|                     o
     S2,Z
  2.025          1.948328457     0.07667154308
  0.612          0.6749003587   ¯0.06290035874
  1.442          1.377723177     0.06427682286
  0.887          0.9662203838   ¯0.07922038384
  0.502          0.3692444914    0.1327555086
  1.115          1.106471471     0.008528528834
  1.126          1.266111661    ¯0.[illegible]01110608
```

```
Z←S3 REGRESS ●(X1,X2,X3,X4,X5*÷5)

                         ANOVA

   SOURCE   DF      SUM SQUARES      MEAN SQUARE        F-RATIO
REGRESSION   5       1.1506E00       2.3012E¯1        1.0929E+1
  RESIDUAL   1       2.1057E¯2       2.1057E¯2
TOTAL        6        1.1717E0

R SQUARE:  0.9820281748
STD ERROR: 0.1451099388
   COEFFICIENTS   T STATISTICS
         2.5303          0.8541
         1.859           2.2104
        ¯0.4898         ¯3.2157
         0.3851          0.4258
         0.3374          0.3251
        ¯1.1279         ¯0.1655
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.409325477
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 1.8
RANGE OF Y: ¯0.1 0.1
```



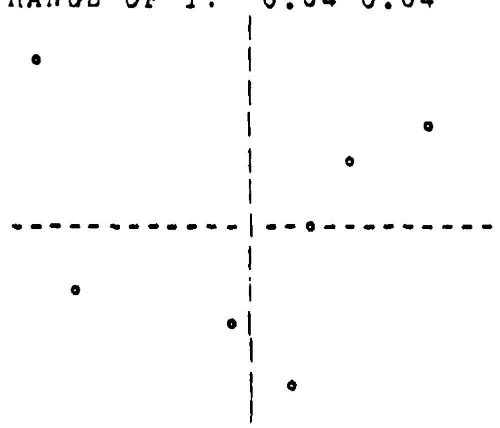
```
      S3,Z
 1.707            1.660630571       0.04636942883
 0.559            0.5970408896     ¯0.03804088963
 1.49             1.451126652       0.03887334783
 0.977            1.024910917      ¯0.04791091717
 0.525            0.444712101       0.08028789902
 1.212            1.206842115       0.005157885114
 1.137            1.221736754      ¯0.08473675399
```

```
Z←S4 REGRESS ⍉(X1,X2,X3,X4,X5*÷5)

                              ANOVA

   SOURCE   DF      SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION   5        1.3672E00       2.7345E¯1          6.4884E00
  RESIDUAL   1        4.2144E¯2       4.2144E¯2
TOTAL        6         1.4094E0

R SQUARE:  0.9700973306
STD ERROR: 0.2052901061
   COEFFICIENTS   T STATISTICS
         0.0927         0.0221
         2.0585         1.73
        ¯0.4563        ¯2.1175
        ¯0.5009        ¯0.3914
        ¯0.4159        ¯0.2833
         5.1473         0.5339
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.409325477
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0 2
RANGE OF Y: ¯0.15 0.15
```



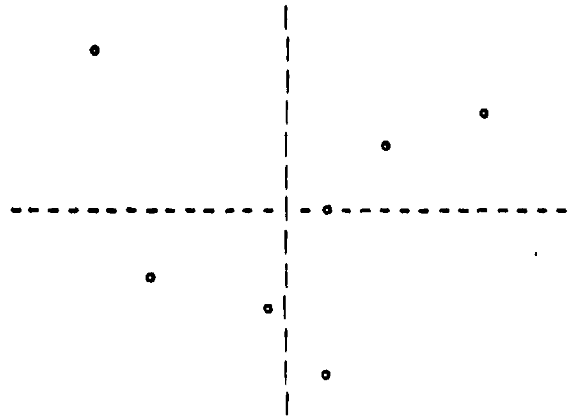
```
      S4,Z
  1.892          1.826400186      0.06559981375
  0.577          0.6308172528    ¯0.05381725282
  1.452          1.397[illegible]5051      0.05499494909
  0.931          0.99878059      ¯0.06778058998
  0.51           0.3964150076     0.1135849924
  1.188          1.180703031      0.007296969393
  1.12           1.239878882     ¯0.1108788818
```

```
Z←(⍟S1) REGRESS ⍟(X1,X2,X3,X4,X5*÷5)

                              ANOVA

    SOURCE  DF      SUM SQUARES       MEAN SQUARE          F-RATIO
REGRESSION   5        1.3959E00        2.7917E¯1          9.3742E+1
  RESIDUAL   1        2.9781E¯3        2.9781E¯3
TOTAL        6         1.3988E0

R SQUARE:  0.9978710193
STD ERROR: 0.05457208236
   COEFFICIENTS   T STATISTICS
         1.6348        1.4672
         2.0404        6.451
        ¯0.5666       ¯9.8921
         0.2512        0.7383
        ¯0.1204       ¯0.3085
        ¯0.2792       ¯0.1089
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.409325477
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: ¯0.8 0.8
RANGE OF Y: ¯0.04 0.04
```



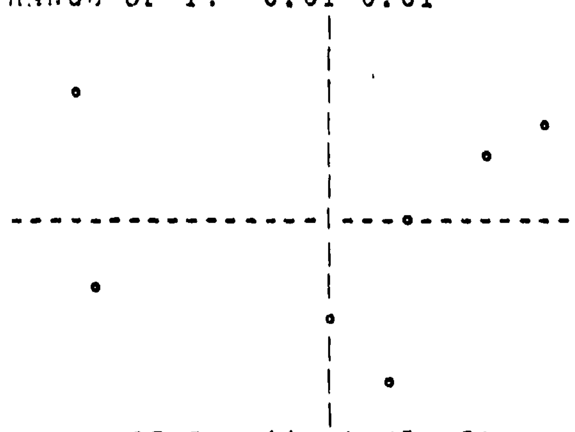
```
      ZI←7 1⍴((*1)*Z[;1])
      S1,ZI,(S1-ZI)
  1.906              1.873050652      0.03294934799
  0.559              0.5670546392    ¯0.008054639232
  1.435              1.414173967      0.02082603281
  0.939              0.9560722943    ¯0.01707229431
  0.51               0.4948311189     0.01516888111
  1.243              1.240591232      0.002408768246
  1.102              1.137683304     ¯0.03568330369
      SSI←(S1-ZI)*2
      SF←+/1 7⍴SSI
      SF
0.003384918903
```

```
      Z←(⍉S2) REGRESS ⍉(X1,X2,X3,X4,X5*÷5)

                              ANOVA

    SOURCE  DF      SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION   5       1.3861E00        2.7722E¯1        1.3629E+2
  RESIDUAL   1       2.0340E¯3        2.0340E¯3
TOTAL        6        1.3882E0

R SQUARE:  0.998534731
STD ERROR: 0.04510013801
   COEFFICIENTS    T STATISTICS
         0.2476          0.2688
         2.2146          8.4723
        ¯0.4667         ¯9.858
        ¯0.0764         ¯0.2718
        ¯0.0462         ¯0.1433
         2.0998          0.9914
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.409325477
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: ¯1 1
RANGE OF Y: ¯0.03 0.03
```



```
      ZI←7 1⍴((*1)*Z[;1])
      S2,ZI,(S2-ZI)
  2.025               1.996025777      0.02897422331
  0.612               0.6192786809    ¯0.007278680929
  1.442               1.424682826      0.01731717406
  0.887               0.9003068819    ¯0.01330688194
  0.502               0.4896283617     0.0123716383
  1.115               1.113214009      0.001795990679
  1.126               1.156048466     ¯0.03004846575
      SSI←(S2-ZI)*2
      SE←+/1 7⍴SSI
      SE
0.002428599928
```

```
Z←(⍉S3) REGRESS ⍉(X1,X2,X3,X4,X5*÷5)

                              ANOVA

    SOURCE   DF      SUM SQUARES       MEAN SQUARE          F-RATIO
REGRESSION    5        1.2522E00        2.5045E¯1          1.4382E+3
  RESIDUAL    1        1.7414E¯4        1.7414E¯4
TOTAL         6         1.2524E0

R SQUARE:  0.9998609606
STD ERROR: 0.01319604426
   COEFFICIENTS   T STATISTICS
         2.8103        10.4302
         1.7547        22.9416
        ¯0.5895       ¯42.5634
         0.8029         9.7613
         0.5075         5.3767
        ¯3.939         ¯6.3564
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.409325477
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: ¯0.8 0.6
RANGE OF Y: ¯0.01 0.01
```

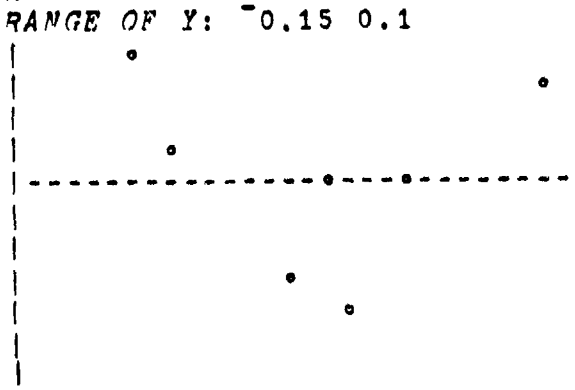


```
      ZI←7 1⍴((⌹1)+Z[;1])
      S3,ZI,(S3-ZI)
   1.707          1.699817154     0.007182845697
   0.559          0.5609371377   ¯0.001937137671
   1.49           1.484742038     0.00525796162
   0.977          0.9812660122   ¯0.004266012197
   0.525          0.5211808077    0.003819192226
   1.212          1.211431646     0.000568354156
   1.137          1.145795353    ¯0.008795352767
      SSI←(S3-ZI)*2
      SE←+/1 7⍴SSI
      SE
0.0001934582814
```

```
      Z←(⍟S4) REGRESS ⍟(X1,X2,X3,X4,X5*÷5)

                              ANOVA

    SOURCE   DF      SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION    5       1.3476E00        2.6952E¯1          1.7886E+2
  RESIDUAL    1       1.5068E¯3        1.5068E¯3
TOTAL         6        1.3491E0

R SQUARE:  0.998883066
STD ERROR: 0.03881813374
   COEFFICIENTS   T STATISTICS
         1.4651          1.8485
         2.0279          9.0135
        ¯0.5382        ¯13.2096
         0.2827          1.1685
         0.0741          0.2668
        ¯0.4211         ¯0.231
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.409325477
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: ¯0.8 0.8
RANGE OF Y: ¯0.03 0.03
            |
            |
  o         |
            |
            |
            |    o   o
            |
------------|--o----------
            |
    o       |
            |
           o|
            |
            | o
            |
      ZI←7 1⍴((*1)*Z[;1])
      S4,ZI,(S4-ZI)
  1.892           1.868676184        0.02332381644
  0.577           0.5829016791      ¯0.0059016791
  1.452           1.436978963        0.0150210367
  0.931           0.9430090241      ¯0.01200902406
  0.51            0.4991631693       0.01083683069
  1.188           1.186361954        0.001638045713
  1.12            1.145677864       ¯0.02567786354
      SSI←(S4-ZI)*2
      SE←+/1 7⍴SSI
      SE
0.001728151201
```

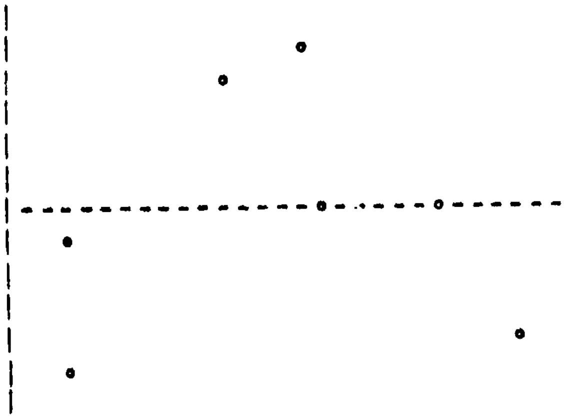
```
Z←S1 REGRESS 1↓(X1,X2,X3,X4,X5*÷5)

                              ANOVA

   SOURCE   DF      SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION   5       1.4355E00        2.8710E¯1          2.1282E+1
  RESIDUAL   1       1.3490E¯2        1.3490E¯2
TOTAL        6        1.4490E0

R SQUARE:  0.9906900953
STD ERROR: 0.1161471181
   COEFFICIENTS    T STATISTICS
         2.069           0.6726
        ¯1.3901         ¯2.6827
         8.6761          3.9504
        ¯6.1385         ¯0.7736
        ¯0.1118         ¯0.0899
         1.3895          0.1577
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.851122569
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.08 0.06
|∘
|                    ∘
|
|
|
| ∘
|-----------∘--∘---------
|
|
|
|
|       ∘
|
|          ∘
|

    S1,Z
  1.906             1.852462984       0.05353701608
  0.559             0.5479603354      0.0000329604597
  1.435             1.435210564      ¯0.000210563F372
  0.939             0.9933989501     ¯0.05439895007
  0.51              0.4532124921      0.05678750793
  1.243             1.241750419       0.001249581086
  1.102             1.168004196      ¯0.06600419598
```

```
Z←S2 REGRESS 1↓(X1,X2,X3,X4,X5*÷5)

                              ANOVA

   SOURCE    DF      SUM SQUARES       MEAN SQUARE           F-RATIO
REGRESSION    5       1.5820E00         3.1640E¯1          9.7072E00
  RESIDUAL    1       3.2594E¯2         3.2594E¯2
TOTAL         6        1.6146E0

R SQUARE:  0.9798127011
STD ERROR: 0.1805389721
   COEFFICIENTS    T STATISTICS
        4.8461          1.0135
       ¯1.5432         ¯1.916
        7.5585          2.2141
       ¯1.0158         ¯0.0824
       ¯0.3269         ¯0.1691
       ¯4.9768         ¯0.3633
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.851122569
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0 2
RANGE OF Y: ¯0.15 0.1
```



```
        S2,Z
  2.025            1.941782112       0.08321788788
  0.612            0.597948847       0.01405115296
  1.442            1.4423273        ¯0.00032729992
  0.887            0.9715576773     ¯0.08455767727
  0.502            0.4137295506      0.08827044937
  1.115            1.113057652       0.001942347676
  1.126            1.223596861      ¯0.1025368607
```

```
      Z←S3 REGRESS 1⍉(X1,X2,X3,X4,X5*⍉5)

                                    ANOVA

    SOURCE  DF       SUM SQUARES        MEAN SQUARE             F-RATIO
REGRESSION   5        1.1716E00          2.3432E¯1           5.7800E+3
  RESIDUAL   1        4.0541E¯5          4.0541E¯5
TOTAL        6        1.1717E0

R SQUARE:  0.9999653991
STD ERROR: 0.006367144447
   COEFFICIENTS   T STATISTICS
        ¯0.6995        ¯4.1479
        ¯1.0298       ¯36.2544
         9.1496        75.9947
       ¯13.6186       ¯31.3075
        ¯1.2258       ¯17.9807
         9.2415        19.131
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.851122569
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 1.8
RANGE OF Y: ¯0.004 0.004
```



```
      S3,Z
  1.707              1.709934881         ¯0.002934880659
  0.559              0.559495548         ¯0.00049554799
  1.49               1.489988457          0.00001154302513
  0.977              0.9740178684         0.00298213158
  0.525              0.5281130715        ¯0.003113071494
  1.212              1.212068502         ¯0.00006850160229
  1.137              1.133381673          0.003618327137
```

```
Z←S4 REGRESS 1+(X1,X2,X3,X4,X5*÷5)

                              ANOVA

   SOURCE   DF      SUM SQUARES      MEAN SQUARE        F-RATIO
REGRESSION   5       1.3986E00        2.7971E¯1        2.3866E+1
  RESIDUAL   1       1.0814E¯2        1.0814E¯2
TOTAL        6        1.4094E0

R SQUARE:  0.9923271201
STD ERROR: 0.1039901585
   COEFFICIENTS   T STATISTICS
         2.2872        0.8304
        ¯1.3442       ¯2.8974
         8.4112        4.2775
        ¯6.3906       ¯0.8995
        ¯0.4979       ¯0.4472
         1.3043        0.1653
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.851122569
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.06 0.06
|
|o                    o
|
|
|
|  o
|-----------o---o---------
|
|
|
|
|        o
|           o
     S4,Z
  1.892          1.84406663        0.04793337006
  0.577          0.5689065584      0.008093441584
  1.452          1.452188524      ¯0.0001885242314
  0.931          0.9797050866     ¯0.0487050866
  0.51           0.4591563626      0.05084363738
  1.188          1.186881211       0.001118789147
  1.12           1.173095627      ¯0.05309562735
```

```
      Z←(1↓J1) REGRESS X1,X2,X3,X4,X5*÷5

                              ANOVA

   SOURCE   DF       SUM SQUARES       MEAN SQUARE          F-RATIO
REGRESSION   5         1.8343E00        3.6686E¯1         7.5089E+3
  RESIDUAL   1         4.8858E¯5        4.8858E¯5
TOTAL        6          1.8344E0

R SQUARE:  0.9999733655
STD ERROR: 0.006989823635
   COEFFICIENTS   T STATISTICS
        ¯1.7183        ¯2.9816
        ¯3.4624       ¯28.0717
         0.0362        45.9891
        ¯0.0604        ¯8.4164
        ¯0.2004        ¯3.9998
         2.7103         7.1516
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.555438004
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.5 2
RANGE OF Y: ¯0.004 0.006
|
|
|   o
|                                o
|
|
o----o-----------------------
|         o
|                            o
|
|        o
      ZI←7 1ρ((Z*¯1)[;1])
      S1,ZI,(S1-ZI)
  1.906               1.905568179         0.0004318206845
  0.559               0.5582614356        0.0007385644014
  1.435               1.443596702        ¯0.008596702468
  0.939               0.9376920903        0.001307909686
  0.51                0.5108032046       ¯0.000803204616
  1.243               1.2437558          ¯0.0007557996165
  1.102               1.097463582         0.004536418307
      SSI←(S1-ZI)*2
      SE←+/1 7ρSSI
      SE
0.00009814132932
```

```
      Z←(1↓S2) REGRESS X1,X2,X3,X4,X5*÷5

                                  ANOVA

     SOURCE  DF       SUM SQUARES      MEAN SQUARE            F-RATIO
 REGRESSION   5        1.6947E00        3.3894E¯1          5.9754E+1
   RESIDUAL   1        5.6723E¯3        5.6723E¯3
 TOTAL        6         1.7004E0

 R SQUARE:  0.9966641189
 STD ERROR: 0.07531465437
    COEFFICIENTS    T STATISTICS
          ¯3.1199         ¯0.5024
          ¯4.3948         ¯3.3069
          _0.0251         _2.9624
          ¯0.0858         ¯1.1094
          ¯0.533          ¯0.9873
           4.1525          1.0169
 DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
 N
 DURBIN-WATSON: 2.555438
 DO YOU WANT TO FORECAST A VALUE FOR Y?
 N
 DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
 Y
 RANGE OF X: 0 2
 RANGE OF Y: ¯0.06 0.06
 |
 |
 |        o
 |                            o
 |
 |           o
 |------o----------------------
 |
 |               o
 |                      o
 |            o
 |
 |
      ZI←7 1ρ((Z*¯1)[;1])
      S2,ZI,(S2-ZI)
   2.025          2.019760466       0.005239533573
   0.612          0.6025956641     _0.009404335855
   1.442          1.541385353      ¯0.09938535324
   0.887          0.8745837991      0.01241620089
   0.502          0.5105138389     ¯0.008513838947
   1.115          1.121587494      ¯0.006587494311
   1.126          1.076988001       0.04901139874
      SSI←(S2-ZI)*2
      SE←+/1 7ρSSI
      SE
 0.0126655613
```

```
      Z←(1↓S3) REGRESS X1,X2,X3,X4,X5*÷5

                                ANOVA

    SOURCE  DF      SUM SQUARES       MEAN SQUARE          F-RATIO
REGRESSION   5       1.7004E00         3.4009E¯1         4.3896E+3
  RESIDUAL   1       7.7475E¯5         7.7475E¯5
TOTAL        6        1.7005E0

R SQUARE:  0.99995444
STD ERROR: 0.008802009025
   COEFFICIENTS   T STATISTICS
        ¯4.8033       ¯6.6186
        ¯3.4655      ¯22.3121
         0.0343       34.6241
        ¯0.0995      ¯11.0133
        ¯0.5196       ¯8.2353
         4.5826        9.6025
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.555437995
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS  PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.006 0.006
|
|      o
|
|                    o
|        o
|
|--o---------------------
|     o
|
|
|                      o
|   o
|
     ZI←7 1ρ((Z*¯1)[;1])
     S3,ZI,(S3-ZI)
  1.707             1.707436364      ¯0.0004363639887
  0.559             0.5599328282     ¯0.0009328281771
  1.49              1.478487937       0.01151206347
  0.977             0.9787887532     ¯0.001788753224
  0.525             0.5239320481      0.001067951892
  1.212             1.211096357       0.0009036427985
  1.137             1.143139259      ¯0.006139259393
     SSI←(S3-ZI)*2
     SE←+/1 7ρSSI
     SE
0.0001764354227
```

```
      Z←(1÷S4) REGRESS X1,X2,X3,X4,X5*÷5

                                ANOVA

    SOURCE   DF       SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION    5         1.7468E00        3.4937E¯1        4.0250E+2
  RESIDUAL    1         8.6799E¯4        8.6799E¯4
TOTAL         6          1.7477E0

R SQUARE:  0.9995033557
STD ERROR: 0.02946168207
   COEFFICIENTS     T STATISTICS
        ¯3.1456          ¯1.295
        ¯3.8227          ¯7.3531
         0.0317           9.5566
        ¯0.081           ¯2.6779
        ¯0.4106          ¯1.9443
         3.7913           2.3735
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.555438
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.5 2
RANGE OF Y: ¯0.02 0.02
|
|  o                      o
|
|
|o----o---------------------
|       o
|                   o
|    o
|
      ZI←7 1⍴((Z*¯1)[;1])
      S4,ZI,(S4-ZI)
  1.892            1.890207841      0.001792159146
  0.577            0.5736979175     0.003302082528
  1.452            1.489838136     ¯0.03783813559
  0.931            0.9256046821     0.005395317879
  0.51             0.5134026762    ¯0.003402676222
  1.188            1.19091533      ¯0.02915329706
  1.12             1.100513022      0.01948697815
      SSI←(S4-ZI)*2
      SE←+/1 7⍴SSI
      SE
0.001874769213
```

```
      Z←(S1*¯2) REGRESS X1,X2,X3,X4,X5*5

                              ANOVA

    SOURCE  DF      SUM SQUARES      MEAN SQUARE         F-RATIO
REGRESSION   5       3.8638E¯1        7.7275E¯2        1.9962E+2
  RESIDUAL   1       3.8711E¯4        3.8711E¯4
TOTAL        6       3.8676E¯1

R SQUARE:  0.9989991025
STD ERROR: 0.01967510062
   COEFFICIENTS   T STATISTICS
        2.5452         1.5689
       ¯1.2153        ¯3.5005
        0.0184         8.3058
        0.006          0.2974
        0.1161         0.8235
       ¯0.511         ¯0.479
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.555437997
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.6 1.6
RANGE OF Y: ¯0.015 0.015
|
|
|        o
|                 o
|          o
|
|--o-------------------------
|      o
|
|                   o
|
|     o
|
      ZI←7 1ρ((Z*¯2)[;1])
      S1,ZI,(S1-ZI)
  1.906           1.907762466     ¯0.001762465789
  0.559           0.5646103533    ¯0.005610353285
  1.435           1.395667686      0.03933231396
  0.939           0.9466555328    ¯0.007655532789
  0.51            0.503736542      0.006263457959
  1.243           1.239194707      0.003805292571
  1.102           1.126840516     ¯0.02484051581
      SSI←(S1-ZI)*2
      SE←+/1 7ρSSI
      SE
0.002310982837
```

```
      Z←(S2*÷¯2) REGRESS X1,X2,X3,X4,X5*÷5

                              ANOVA

    SOURCE   DF       SUM SQUARES        MEAN SQUARE          F-RATIO
REGRESSION    5        3.6847E¯1          7.3694E¯2         1.4674E+3
  RESIDUAL    1        5.0221E¯5          5.0221E¯5
TOTAL         6        3.6852E¯1

R SQUARE:  0.9998637232
STD ERROR: 0.00708668237
   COEFFICIENTS    T STATISTICS
         2.7384         4.6866
        ¯1.4461       ¯11.5643
         0.0147        18.4405
         0.0039         0.5415
         0.0129         0.2536
        ¯0.3913        ¯1.0183
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.555438004
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.6 1.6
RANGE OF Y: ¯0.004 0.006
|
|
|     o
|                o
|
|
|--o-----o------------------
|
|         o    o
|
|       o
      ZI←7 1⍴((Z*¯2)[;1])
      S2,ZI,(S2-ZI)
  2.025              2.024305474        0.0006945256332
  0.612              0.6097088633       0.002291136676
  1.442              1.456682144       ¯0.01468214382
  0.887              0.8844891513       0.002510848688
  0.502              0.5042310488      ¯0.002231048825
  1.115              1.116168047       ¯0.001168046685
  1.126              1.116966985        0.009033015016
      SSI←(S2-ZI)*2
      SE←+/1 7⍴SSI
      SE
0.0003155386536
```

```
      Z←(S3*÷¯2) REGRESS X1,X2,X3,X4,X5*÷5

                              ANOVA

     SOURCE   DF      SUM SQUARES     MEAN SQUAR[illegible]        [illegible]RATIO
 REGRESSION    5        3.5422E¯1       7.0844E¯[illegible]          [illegible]29E+[illegible]
   RESIDUAL    1        6.4823E¯4       6.4823E¯4
 TOTAL         6        3.5487E¯1

 R SQUARE:  0.9981733045
 STD ERROR: 0.02546044814
    COEFFICIENTS   T STATISTICS
         0.0555        0.0264
        ¯1.3965       ¯3.1085
         0.0164        5.7321
        ¯0.0255       ¯0.9744
        ¯0.1223       ¯0.67
         1.0487        0.7597
 DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
 N
 DURBIN-WATSON: 2.555437998
 DO YOU WANT TO FORECAST A VALUE FOR Y?
 N
 DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
 Y
 RANGE OF X: 0.6 1.4
 RANGE OF Y: ¯0.02 0.015
 |
 |          o
 |
 |                         o
 |             o
 |
 |----o----------------------
 |          o
 |
 |
 |                           o
 |       o
 |
 |
      ZI←7 1ρ((Z*¯2)[;1])
      S3,ZI,(S3-ZI)
   1.707          1.708933318     ¯0.001933318086
   0.559          0.5662761163    ¯0.007276116315
   1.49           1.436500502      0.05349949764
   0.977          0.9875345749    ¯0.0053457494
   0.525          0.5165590017     0.008440998275
   1.212          1.207261868      0.004738131874
   1.137          1.17086746      ¯0.03386745976
      SSI←(S3-ZI)*2
      SE←+/1 7ρSSI
      SE
 0.004270558281
```

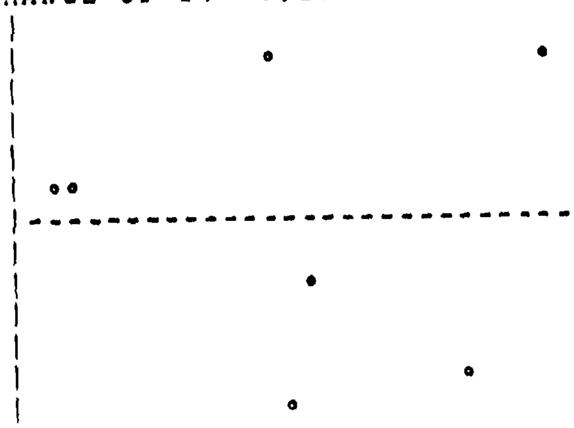
```
      Z←(S4*÷¯2) REGRESS X1,X2,X3,X4,X5*÷5

                              ANOVA

    SOURCE   DF       SUM SQUARES       MEAN SQUARE          F-RATIO
REGRESSION    5        3.7065E¯1         7.4130E¯2         6.2498E+2
  RESIDUAL    1        1.1861E¯4         1.1861E¯4
TOTAL         6        3.7077E¯1

R SQUARE:  0.9996800938
STD ERROR: 0.01089085959
   COEFFICIENTS   T STATISTICS
          1.92         2.1382
         ¯1.3532      ¯7.0413
          0.0166      13.5126
         ¯0.0034      ¯0.3073
          0.0139       0.1783
         ¯0.0334      ¯0.0566
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.555437996
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.6 1.6
RANGE OF Y: ¯0.008 0.006
|       o
|
|                 o
|
|          o
|
|--o----------------------
|       o
|
|
|
|                   o
|     o
|
|
      ZI←7 1ρ((Z*¯2)[;1])
      S4,ZI,(S4-ZI)
  1.892              1.892964558         ¯0.0009645580917
  0.577              0.5802460494        ¯0.03246049377
  1.452              1.429635657          0.02236434331
  0.931              0.9351721303        ¯0.00417213034
  0.51               0.5065186153         0.003481384695
  1.188              1.186029805          0.001970195404
  1.12               1.13398418          ¯0.01398417992
      SSI←(S4-ZI)*2
      SE←+/1 7ρSSI
      SE
0.0007405967294
```

```
Z←S1 REGRESS (2×X1×X4),(4÷X2*1÷2),((X5÷X3)*1÷2)

                              ANOVA

    SOURCE  DF     SUM SQUARES      MEAN SQUARE         F-RATIO
REGRESSION   3       1.3526E00       4.5088E¯1        1.4035E+1
  RESIDUAL   3       9.6378E¯2       3.2126E¯2
TOTAL        6        1.4490E0

R SQUARE:  0.933487063
STD ERROR: 0.1792372431
   COEFFICIENTS    T STATISTICS
         0.5362          0.7998
         0.4507          2.74
         1.1815          3.7876
        ¯0.9973         ¯2.1885
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.492834464
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 1.8
RANGE OF Y: ¯0.2 0.2
|
|             o            o
|
| o
|-o-------------------------
|              o
|
|             o        o
|


     S1,Z
  1.906          1.756426099     0.1495739014
  0.559          0.5210857222    0.03791427776
  1.435          1.561861667    ¯0.1268616666
  0.939          1.103710491    ¯0.164710491
  0.51           0.4932825522    0.01671744784
  1.243          1.086958453     0.156041547
  1.102          1.170675016    ¯0.0686750164
```

```
      Z←S2 REGRESS (2×X1×X4),(4↑X2*1÷2),((X5÷X3)*1÷2)

                              ANOVA

    SOURCE  DF      SUM SQUARES      MEAN SQUARE        F-RATIO
REGRESSION   3       1.4998E00        4.9994E¯1        1.3068E+1
  RESIDUAL   3       1.1477E¯1        3.8258E¯2
TOTAL        6        1.6146E0

R SQUARE:  0.9289143785
STD ERROR: 0.1955969662
   COEFFICIENTS    T STATISTICS
         0.1185          0.162
         0.5471          3.0478
         1.0069          2.9581
        ¯0.847          ¯1.7033
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.296471296
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.2 0.2
|                    o
|          o
|
|o
|--o----------------------
|           o
|
|                o
|          o

     S2,Z
  2.025          1.840810071     0.1841899287
  0.612          0.6011698712    0.01083012877
  1.442          1.608561       ¯0.1665609997
  0.887          1.065525015    ¯0.1785250153
  0.502          0.4477593823    0.05424061769
  1.115          0.9846729568    0.1303270432
  1.126          1.160501703    ¯0.03450170338
```

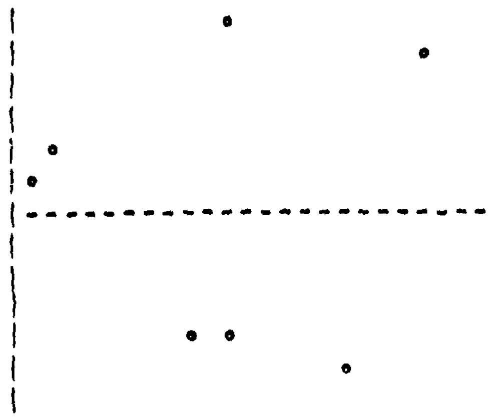
```
Z←S3 REGRESS (2×X1×X4),(4÷X2*1÷2),((X5÷X3)*1÷2)

                              ANOVA

    SOURCE  DF      SUM SQUARES        MEAN SQUARE            F-RATIO
REGRESSION   3        1.1468E00          3.8226E¯1          4.6064E+1
  RESIDUAL   3        2.4895E¯2          8.2984E¯3
TOTAL        6         1.1717E0

R SQUARE:  0.978752256
STD ERROR: 0.09109548491
   COEFFICIENTS   T STATISTICS
         0.8934         2.6222
         0.4386         5.2463
         1.1298         7.1266
        ¯1.1242        ¯4.8538
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.903207432
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 1.8
RANGE OF Y: ¯0.1 0.1
|              o
|
|
|                          o
| o
|
|---------------------------
|  o
|
|            o      o
|
|              o
|

      S3,Z
  1.707          1.651574796     0.05542520401
  0.559          0.520570653     0.03842934701
  1.49           1.534377125    ¯0.04437712544
  0.977          1.053076592    ¯0.07607659174
  0.525          0.544095763    ¯0.01909576301
  1.212          1.114383706     0.09761629381
  1.137          1.188921365    ¯0.05192136465
```

```
Z←S4 REGRESS (2×X1×X4),(4÷X2*1÷2),((X5÷X3)*1÷2)

                         ANOVA

    SOURCE  DF      SUM SQUARES      MEAN SQUARE        F-RATIO
REGRESSION   3       1.3354E00        4.4513E¯1       1.8050E+1
  RESIDUAL   3       7.3982E¯2        2.4661E¯2
TOTAL        6        1.4094E0

R SQUARE:  0.9475074247
STD ERROR: 0.1570367694
   COEFFICIENTS   T STATISTICS
         0.4859        0.8272
         0.4814        3.3405
         1.1057        4.0457
        ¯0.979        ¯2.4521
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.430730445
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 1.8
RANGE OF Y: ¯0.15 0.15
```



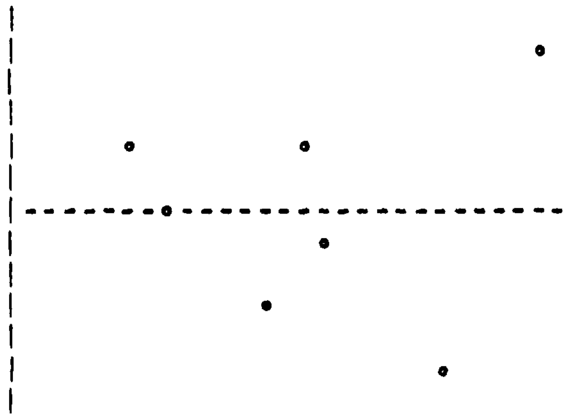
```
      S4,Z
  1.892          1.756691175     0.1353088248
  0.577          0.5484520629    0.02854793706
  1.452          1.57003973     ¯0.1180397295
  0.931          1.075558692    ¯0.1445586918
  0.51           0.490366912     0.01963308797
  1.188          1.057580309     0.1304196909
  1.12           1.171311119    ¯0.05131111946
```

```
Z←S1 REGRESS (((2×X1)*2)×X4),(4+X2*1÷2),((X3+X5)*1÷2)

                              ANOVA

   SOURCE  DF      SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION  3       1.4150E00        4.7167E¯1         4.1616E+1
  RESIDUAL  3       3.4002E¯2        1.1334E¯2
TOTAL       6        1.4490E0

R SQUARE:  0.9765346449
STD ERROR: 0.1064605627
   COEFFICIENTS   T STATISTICS
        ¯2.9736        ¯4.1419
         0.208          5.135
         1.0298         5.458
         4.0495         3.4965
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.709393195
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.1 0.1
```



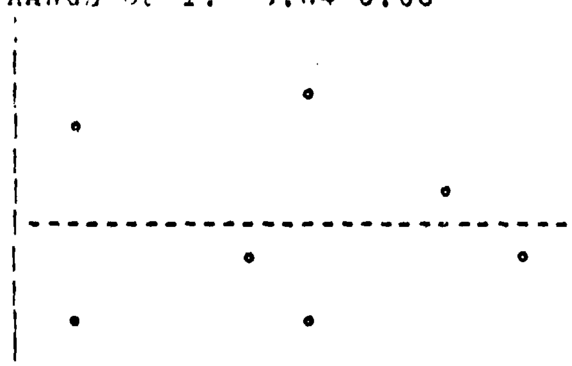
```
      S1,Z
  1.906           1.826263553     0.07973044743
  0.559           0.5253347306   ¯0.03366526936
  1.435           1.5258725      ¯0.09087250019
  0.939           1.00544866     ¯0.06644866008
  0.51            0.4995394704    0.01046052956
  1.243           1.145123686    ¯0.09787631386
  1.102           1.1664114      ¯0.06441139993
```

```
Z←S2 REGRESS (((2×X1)*2)×X4),(4÷X2*1÷2),((X3÷X5)*1÷2)

                        ANOVA

    SOURCE  DF     SUM SQUARES     MEAN SQUARE        F-RATIO
REGRESSION   3       1.5728E00      5.2427E¯1        3.7649E+1
  RESIDUAL   3       4.1776E¯2      1.3925E¯2
TOTAL        6        1.6146E0

R SQUARE:  0.9741257908
STD ERROR: 0.1180061862
   COEFFICIENTS   T STATISTICS
        ¯2.5667       ¯3.2253
         0.2486        5.538
         0.8465        4.0476
         3.2114        2.5016
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.345852334
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0 2
RANGE OF Y: ¯0.15 0.15
```



```
     S2,Z
 2.025            1.912181883      0.1128181174
 0.612            0.6065122656     0.005487734427
 1.442            1.571422061     ¯0.1294220607
 0.887            0.9591245226    ¯0.07212452264
 0.502            0.4498976576     0.05210234236
 1.115            1.055459981      0.05954001311
 1.126            1.15440163      ¯0.02840162998
```

```
Z←S3 REGRESS (((2×X1)*2)×X4),(4÷X2*1÷2),((X3÷X5)*1÷2)

                              ANOVA

    SOURCE   DF       SUM SQUARES        MEAN SQUARE           F-RATIO
REGRESSION    3        1.1663E00          3.8878E¯1          2.1906E+2
  RESIDUAL    3        5.3242E¯3          1.7747E¯3
TOTAL         6         1.1717E0

R SQUARE:  0.9954558534
STD ERROR: 0.04212758929
   COEFFICIENTS    T STATISTICS
         ¯2.9864        ¯10.5121
          0.7899         11.8467
          0.5841         13.1807
          4.3479          9.4872
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.841201446
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 1.8
RANGE OF Y: ¯0.04 0.06
```

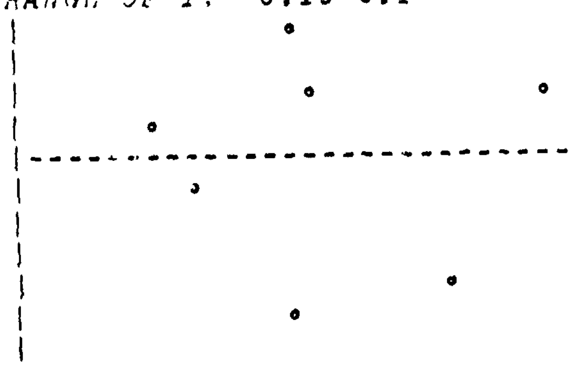


```
    S3,Z
  1.707          1.719449471      ¯0.01244947073
  0.559          0.5311134601      0.02788653989
  1.49           1.478253498       0.0117465021
  0.977          0.9828400917     ¯0.005840091717
  0.525          0.5578202619     ¯0.03282026191
  1.212          1.167041473       0.04495852676
  1.137          1.170481744      ¯0.0348174438
```

```
Z←S4 REGRESS (((2×X1)*2)×X4),(4÷X2*1÷2),((X3÷X5)*1÷2)

                              ANOVA

    SOURCE   DF        SUM SQUARES        MEAN SQUARE          F-RATIO
REGRESSION    3         1.3894E00          4.6314E¯1         6.9615E+1
  RESIDUAL    3         1.9959E¯2          6.6529E¯3
TOTAL         6          1.4094E0

R SQUARE:  0.9858386487
STD ERROR: 0.08156517286
   COEFFICIENTS    T STATISTICS
        ¯2.8327          ¯5.15
         0.2173           7.0017
         0.9526           6.5898
         3.8331           4.3199
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.598984712
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.1 0.1
|
|
|          o         o
|
|
|oo
|-----------------------
|
|
|        o o
|
|              o
|

      S4,Z
  1.892          1.826496706    0.06550329443
  0.577          0.5548634763   0.02213652372
  1.452          1.527993736   ¯0.07599373607
  0.931          0.9823250092  ¯0.05132500924
  0.51           0.4972535752   0.01274642481
  1.188          1.118661263    0.06933873692
  1.12           1.162406235   ¯0.04240623458
```

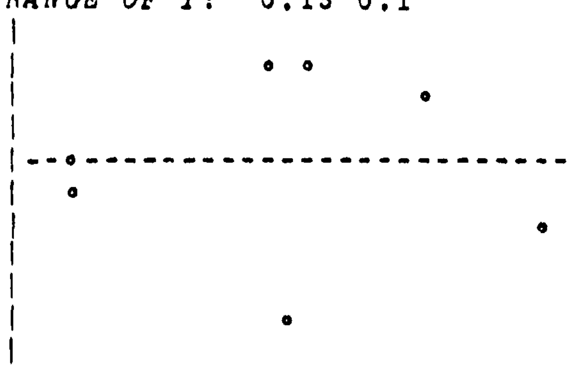
```
      A←((2×X1)*2)×X4
      B←(4÷X2*1÷2)×((X3+X5)*1÷2)

      Z←S1 REGRESS (A*2),B

                              ANOVA

    SOURCE  DF      SUM SQUARES       MEAN SQUARE           F-RATIO
REGRESSION   2        1.4002E00        7.0008E¯1          5.7328E+1
  RESIDUAL   4        4.8847E¯2        1.2212E¯2
TOTAL        6         1.4490E0

R SQUARE:  0.9662893666
STD ERROR: 0.1105068706
   COEFFICIENTS   T STATISTICS
        ¯0.5151         ¯2.5324
         0.0172          5.5232
         2.2495          5.1911
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.421375557
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.2 0.15
|
|          o
|
|
|
|                     o
|o        o
|--o---------------------
|
|
|
|               o
|
|
|         o
|
|

     S1,Z
  1.906            1.864413348     0.04158665157
  0.559            0.5684681973   ¯0.009468197349
  1.435            1.498560859    ¯0.06356085924
  0.939            1.09098087     ¯0.1519808697
  0.51             0.4885860395    0.02141396053
  1.243            1.105836002     0.1371639983
  1.102            1.077154684     0.02484531594
```

```
      Z←S2 REGRESS (A*2),B

                                ANOVA

      SOURCE  DF        SUM SQUARES        MEAN SQUARE          F-RATIO
  REGRESSION   2         1.5706E00         7.8528E¯1          7.1325E+1
    RESIDUAL   4         4.4040E¯2         1.1010E¯2
  TOTAL        6          1.6146E0

  R SQUARE:  0.9727240819
  STD ERROR: 0.1049280381
     COEFFICIENTS    T STATISTICS
          ¯0.3978        ¯2.0597
           0.0221         7.4628
           1.7916         4.3542
  DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
  N
  DURBIN-WATSON: 0.9997641418
  DO YOU WANT TO FORECAST A VALUE FOR Y?
  N
  DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
  Y
  RANGE OF X: 0 2
  RANGE OF Y: ¯0.15 0.1
```



```
        S2,Z
   2.025            1.962851577     0.062148423
   0.612            0.6349461589   ¯0.02294615885
   1.442            1.544825096    ¯0.1028250961
   0.887            1.010161444    ¯0.1231614436
   0.502            0.4707660818    0.03123391823
   1.115            1.016788829     0.09821117103
   1.126            1.068658814     0.05734118627
```

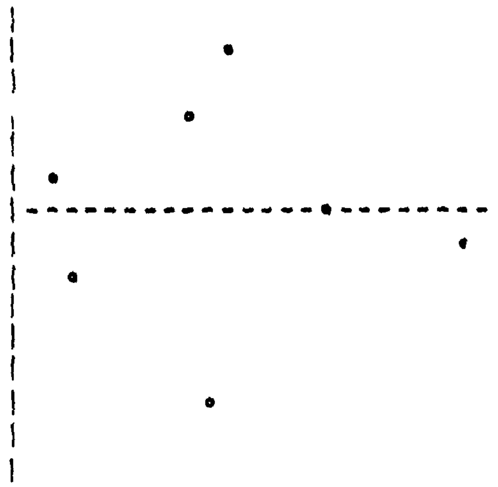
```
Z←S3 REGRESS (A*2),B

                              ANOVA

    SOURCE   DF       SUM SQUARES       MEAN SQUARE          F-RATIO
REGRESSION    2         1.1370E00        5.6848E¯1         6.5517E+1
  RESIDUAL    4         3.4707E¯2        8.6768E¯3
TOTAL         6          1.1717E0

R SQUARE:  0.9703776962
STD ERROR: 0.0931494911
   COEFFICIENTS   T STATISTICS
        ¯0.4412         ¯2.5735
         0.0139          5.2997
         2.2448          6.1457
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.890048191
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 1.8
RANGE OF Y: ¯0.15 0.1
```



```
      S3,Z
  1.707          1.752415297    ¯0.04541529724
  0.559          0.5741186837   ¯0.01511868371
  1.49           1.436550027     0.05344997335
  0.977          1.111465189    ¯0.1344651895
  0.525          0.533491179    ¯0.00849117901
  1.212          1.128312553     0.08368744727
  1.137          1.070647071     0.06635292882
```

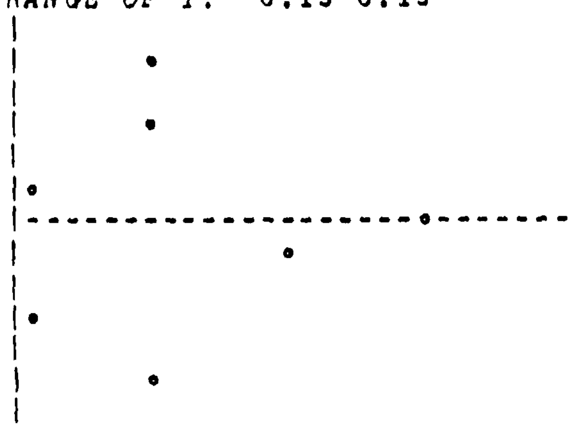
```
      Z←S4 REGRESS (A*2),B


                                   ANOVA

    SOURCE  DF       SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION   2        1.3736E00        6.8681E¯1        7.6817E+1
  RESIDUAL   4        3.5763E¯2        8.9407E¯3
TOTAL        6         1.4094E0

R SQUARE:  0.974624903
STD ERROR: 0.09455552257
   COEFFICIENTS    T STATISTICS
         ¯0.4544          ¯2.6107
          0.018            6.7579
          2.0873           5.6295
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.289508585
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.15 0.15
|
|
|         o
|
|         o
|o                    o
|-----------------------
|  o
|              o
|
|
|         o
|

     S4,Z
  1.892          1.867481112     0.02451888817
  0.577          0.592610303    ¯0.01561030298
  1.452          1.497021268    ¯0.0450212684
  0.931          1.06743801     ¯0.1364380095
  0.51           0.4938865991    0.01611340094
  1.188          1.079949072     0.1080509277
  1.12           1.071613636     0.04838636407
```

```
Z←S1 REGRESS (A*4),B

                         ANOVA

    SOURCE  DF      SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION   2        1.3977E00        6.9883E¯1        5.4427E+1
  RESIDUAL   4        5.1358E¯2        1.2840E¯2
TOTAL        6         1.4490E0

R SQUARE:  0.9645561693
STD ERROR: 0.1133120641
   COEFFICIENTS   T STATISTICS
        ¯0.3209        ¯1.4751
         0.0003         5.3683
         2.2773         5.1505
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.425988595
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.2 0.15
```



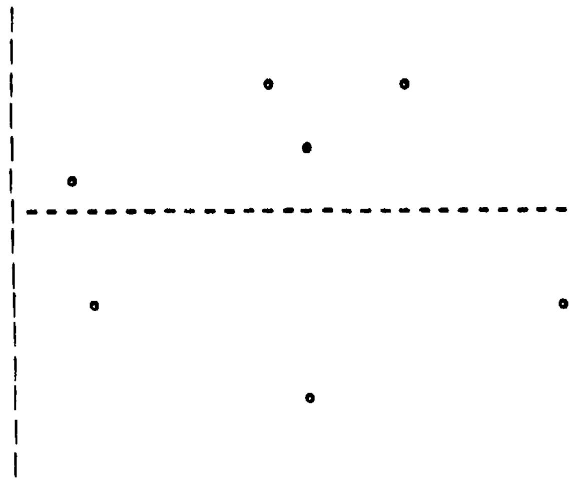
```
      S1,Z
  1.906            1.318834276      ¯0.0128342755
  0.559            0.5291614045      0.02983859555
  1.435            1.436167839      ¯0.001167838731
  0.939            1.098222515      ¯0.1592225153
  0.51             0.56889458       ¯0.05889458
  1.243            1.119243692       0.1237563075
  1.102            1.023475694       0.07852430642
```

```
Z←S2 REGRESS (A*4),B

                                   ANOVA

    SOURCE   DF      SUM SQUARES       MEAN SQUARE          F-RATIO
REGRESSION    2       1.5681E00         7.8404E¯1         6.7412E+1
  RESIDUAL    4       4.6522E¯2         1.1630E¯2
TOTAL         6        1.6146E0

R SQUARE:  0.971186688
STD ERROR: 0.1078446125
   COEFFICIENTS   T STATISTICS
       ¯0.148          ¯0.7147
        0.0003          7.2462
        1.8252          4.3374
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 0.9632304298
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.5 2.5
RANGE OF Y: ¯0.15 0.15
```



```
     S2,Z
  2.025          2.033437915     ¯0.008437915356
  0.612          0.5847001608     0.02729983923
  1.442          1.464986906     ¯0.02298690615
  0.887          1.01900929      ¯0.1320092898
  0.502          0.5737141215    ¯0.07171412153
  1.115          1.033515936      0.0814840643
  1.126          0.9996356707     0.1263643293
```

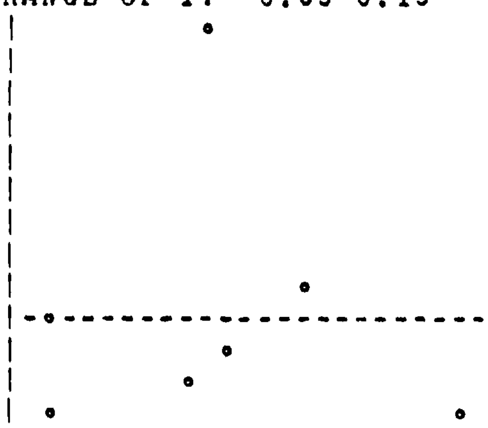
```
Z←S3 REGRESS (A*4),R

                              ANOVA

   SOURCE  DF      SUM SQUARES        MEAN SQUARE            F-RATIO
REGRESSION   2       1.1108E00         5.5540E¯1          3.6503E+1
  RESIDUAL   4       6.0860E¯2         1.5215E¯2
TOTAL        6        1.1717E0

R SQUARE:  0.9480564833
STD ERROR: 0.1233493159
   COEFFICIENTS    T STATISTICS
        ¯0.3001        ¯1.2674
         0.0002         3.7813
         2.3164         4.8127
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.810947154
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 1.8
RANGE OF Y: ¯0.2 0.15
```



```
     S3,Z
  1.707          1.777880095     ¯0.07088009515
  0.559          0.5379032577     0.02109674234
  1.49           1.381032527      0.1089674727
  0.977          1.128032676     ¯0.1510326762
  0.525          0.6005207403    ¯0.07552074029
  1.212          1.150628278      0.06137172232
  1.137          1.030396426      0.1066035743
```

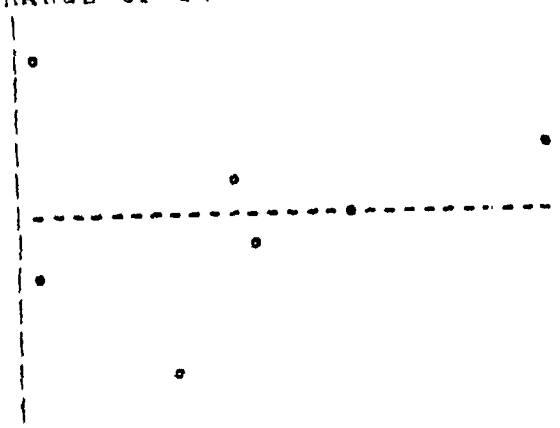
```
      Z←S4 REGRESS (A*4),B

                              ANOVA

    SOURCE  DF      SUM SQUARES      MEAN SQUARE           F-RATIO
REGRESSION   2       1.3622E00        6.8108E¯1          5.7709E+1
  RESIDUAL   4       4.7208E¯2        1.1802E¯2
TOTAL        6        1.4094E0

R SQUARE:  0.9665043083
STD ERROR: 0.1086369388
   COEFFICIENTS   T STATISTICS
        ¯0.2553        ¯1.2243
         0.0003         5.7989
         2.1296         5.0238
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.336659174
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.15 0.15
|
|
|        ∘∘
|
|
|
| ∘            ∘
|-----------------------
|                      ∘
|
|  ∘
|
|
|        ∘

     S4,Z
  1.892          1.919478335   ¯0.02747833468
  0.577          0.5502736899   0.02672631014
  1.452          1.430344515    0.02165548527
  0.931          1.077893149   ¯0.1468931486
  0.51           0.5785205106  ¯0.06852051062
  1.188          1.097063809    0.0909361912
  1.12           1.016425993    0.1035740072
```

```
AA←(X1×4)×((1↓X2)*1↓4)×((1↓X5)*1↓5)
BB←(X3×X4)*1↓3

Z←S1 REGRESS (AA*2),BB

                              ANOVA

    SOURCE    DF       SUM SQUARES       MEAN SQUARE          F-RATIO
REGRESSION     2        1.4223E00         7.1115E¯1         1.0654E+2
  RESIDUAL     4        2.6701E¯2         6.6753E¯3
TOTAL          6         1.4490E0

R SQUARE:  0.9815729219
STD ERROR: 0.08170225859
   COEFFICIENTS    T STATISTICS
        ¯1.4131         ¯4.0925
         2.0433         14.3735
         0.4585          4.6665
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.56431262
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.05 0.15
```



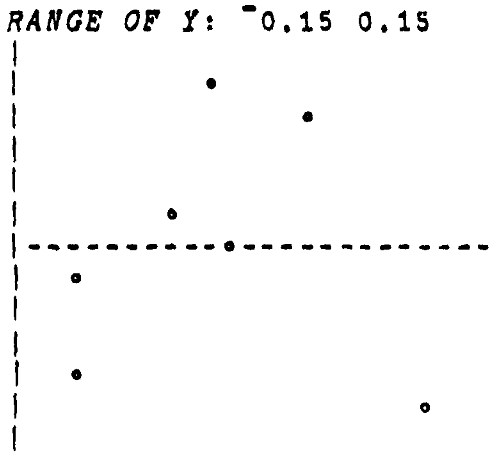
```
      S1,Z
  1.906              1.947833977       ¯0.04183397746
  0.559              0.5623501076      ¯0.003350107601
  1.435              1.423102667        0.01189733331
  0.939              0.9778221447      ¯0.03882214471
  0.51               0.5573728049      ¯0.04737280489
  1.243              1.09986579         0.14313421
  1.102              1.125652509       ¯0.02365250867
```

```
Z←S2 REGRESS (AA*2),BB

                              ANOVA

    SOURCE   DF       SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION    2        1.5997E00         7.9987E¯1         2.1548E+2
  RESIDUAL    4        1.4849E¯2         3.7121E¯3
TOTAL         6         1.6146E0

R SQUARE:  0.9908035651
STD ERROR: 0.06092725545
   COEFFICIENTS    T STATISTICS
        ¯1.6193        ¯6.289
         2.1716        20.4843
         0.5039         6.8772
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.427333649
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.5 2
RANGE OF Y: ¯0.1 0.1
```



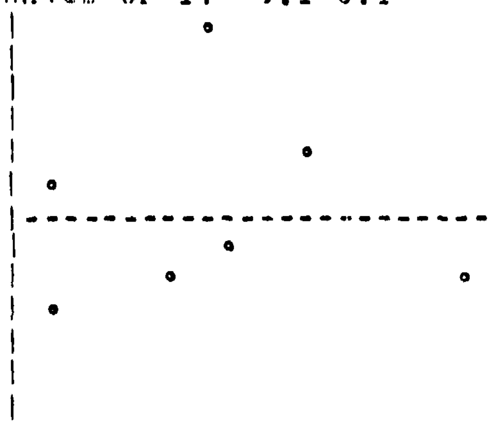
```
      S2,Z
  2.025            1.998140985      0.02685901544
  0.612            0.5280021314     0.08379786856
  1.442            1.448145766     ¯0.06145766455
  0.887            0.9634341807    ¯0.07643418068
  0.502            0.5303481005    ¯0.02834810046
  1.115            1.10117361       0.01382638766
  1.126            1.139695226     ¯0.01369522607
```

```
Z←S3 REGRESS (AA*2),BB

                              ANOVA

    SOURCE   DF      SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION    2        1.1186E00        5.5928E¯1        4.2128E+1
  RESIDUAL    4        5.3102E¯2        1.3276E¯2
TOTAL         6         1.1717E0

R SQUARE:  0.9546777874
STD ERROR: 0.1152196689
   COEFFICIENTS    T STATISTICS
        ¯1.3502        ¯2.7728
         1.827          9.1131
         0.4724         3.4094
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.214813524
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.15 0.15
```



```
      S3,Z
  1.707          1.826353334    ¯0.1193533338
  0.559          0.5962340839   ¯0.03723408393
  1.49           1.386023981     0.1039760191
  0.977          0.9449404918    0.0320595082
  0.525          0.6204337106   ¯0.09543371057
  1.212          1.084050349     0.1279496514
  1.137          1.14890405     ¯0.01190405034
```

```
Z←S4 REGRESS (AA*2),BB

                          ANOVA

   SOURCE   DF      SUM SQUARES       MEAN SQUARE          F-RATIO
REGRESSION   2        1.3937E00        6.9683E¯1         1.7748E+2
  RESIDUAL   4        1.5705E¯2        3.9262E¯3
TOTAL        6         1.4094E0

R SQUARE:  0.9888569863
STD ERROR: 0.06265913215
   COEFFICIENTS   T STATISTICS
        ¯1.4693         ¯5.5487
         2.0289         18.6091
         0.4784          6.3486
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.645165667
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.1 0.1
```



```
      S4,Z
  1.892          1.931289602    ¯0.03928960226
  0.577          0.5588261138    0.01817388625
  1.452          1.420953553     0.03104644683
  0.931          0.9629232927   ¯0.03192329269
  0.51           0.5645143817   ¯0.05451438173
  1.188          1.095206397     0.09279360326
  1.12           1.13628666     ¯0.01628665966
```

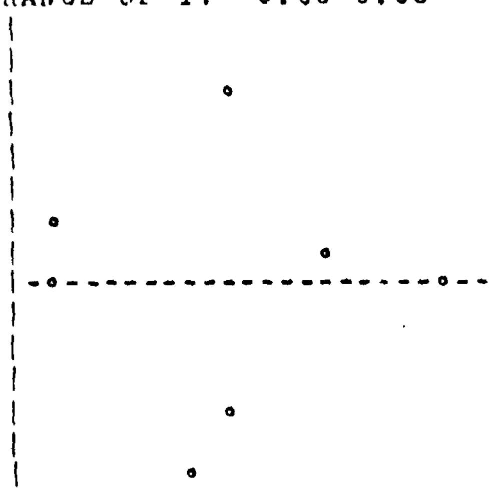
```
      Z←S1 REGRESS (AA*3↑2),BB

                              ANOVA

    SOURCE  DF       SUM SQUARES      MEAN SQUARE           F-RATIO
REGRESSION   2        1.4296E00        7.1478E¯1          1.4697E+2
  RESIDUAL   4        1.9453E¯2        4.8633E¯3
TOTAL        6         1.4490E0

R SQUARE:  0.986574787
STD ERROR: 0.06973750666
   COEFFICIENTS   T STATISTICS
       ¯1.7842        ¯5.7754
        2.3688        16.8837
        0.4622         5.5072
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.304473244
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 2
RANGE OF Y: ¯0.1 0.15
|
|         •
|
|
|
|
|
|-2-------------------•-
|             •
|         •
|        •
|

      S1,Z
  1.906          1.904334068      0.001665932428
  0.559          0.5555183395     0.003481660541
  1.435          1.448975335     ¯0.01397533464
  0.939          1.004620418     ¯0.0656204184
  0.51           0.506383762      0.003616238014
  1.243          1.12879516       0.1142048403
  1.102          1.145372918     ¯0.04337291824
```

```
Z←S2 REGRESS (AA*3⍴2),BB

                          ANOVA

   SOURCE  DF      SUM SQUARES       MEAN SQUARE         F-RATIO
REGRESSION  2       1.5867E00         7.9335E¯1         1.1373E+2
  RESIDUAL  4       2.7904E¯2         6.9760E¯3
TOTAL       6       1.6146E0

R SQUARE:  0.9827177035
STD ERROR: 0.08352232771
   COEFFICIENTS   T STATISTICS
        ¯1.9881        ¯5.3731
         2.5003        14.88
         0.5029         5.0031
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 1.361783889
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0 2
RANGE OF Y: ¯0.15 0.1
|
|      o                   o
|
|      o
|----------------------------
|               2     o
|
|
|             o
|
|

    S2,Z
 2.025          1.946574227     0.07842577321
 0.612          0.5249410664    0.08705893358
 1.442          1.472866743    ¯0.03086674309
 0.887          0.9935040853   ¯0.1065040853
 0.502          0.4799842648    0.02201573524
 1.115          1.131788352    ¯0.01678835242
 1.126          1.159341261    ¯0.03334126125
```

```
Z←S3 REGRESS (AA*3+2),BB

                          ANOVA

    SOURCE  DF      SUM SQUARES      MEAN SQUARE           F-RATIO
REGRESSION   2        1.1439E00        5.7196E¯1         8.2469E+1
  RESIDUAL   4        2.7742E¯2        6.9355E¯3
TOTAL        6         1.1717E0

R SQUARE:  0.9763226687
STD ERROR: 0.08327937299
   COEFFICIENTS   T STATISTICS
        ¯1.71          ¯4.6349
         2.1366        12.7524
         0.4811         4.8
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
N
DURBIN-WATSON: 2.22229438
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.4 1.8
RANGE OF Y: ¯0.1 0.15
|
|
|             o
|                  o
|
|
|--------------------------
|   o          o
|  o
|                        o
|
```

```
      S3,Z
 1.707         1.793256787    ¯0.08625678731
 0.559         0.5855634384   ¯0.02656343842
 1.49          1.412172540     0.07782745396
 0.977         0.9672395632    0.009760436753
 0.525         0.5707474942   ¯0.04574749424
 1.212         1.110058275     0.1019417251
 1.137         1.167961896    ¯0.0309618958
```

```
      Z←S4 REGRESS (AA*3+2),BB

                                  ANOVA

     SOURCE  DF      SUM SQUARES       MEAN SQUARE            F-RATIO
 REGRESSION   2        1.3999E00         6.9994E¯1          2.9474E+2
   RESIDUAL   4        9.4990E¯3         2.3748E¯3
 TOTAL        6         1.4094E0

 R SQUARE:  0.9932601078
 STD ERROR: 0.04873147146
    COEFFICIENTS   T STATISTICS
         ¯1.8366         ¯8.5076
          2.3512         23.9822
          0.4818          8.2156
 DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
 N
 DURBIN-WATSON: 2.36840413
 DO YOU WANT TO FORECAST A VALUE FOR Y?
 N
 DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
 Y
 RANGE OF X: 0.4 2
 RANGE OF Y: ¯0.06 0.08
```



```
      S4,Z
  1.892          1.887845824      0.004154175918
  0.577          0.5522409586     0.02475904142
  1.452          1.446511986      0.005488013759
  0.931          0.9896040021    ¯0.8860400208
  0.51           0.5140669951    ¯0.004066995139
  1.188          1.123924751      0.06407524895
  1.12           1.155805483     ¯0.03580548282
```

APPENDIX D

NORMALITY PLOTS

"All Possible Subsets Regression" was applied to the best equation, number ((13)), to check the assumption about the residuals being normally distributed with mean zero and variance σ^2. BMDP9R [7] was used as program package.

Figures 8 through 11 show normal probability plots for standardized residuals for Groups 1 through 4. If the assumption about normality was met, the standardized residuals versus the expected normal values would follow a straight line. This is however not the case for either of the four groups.

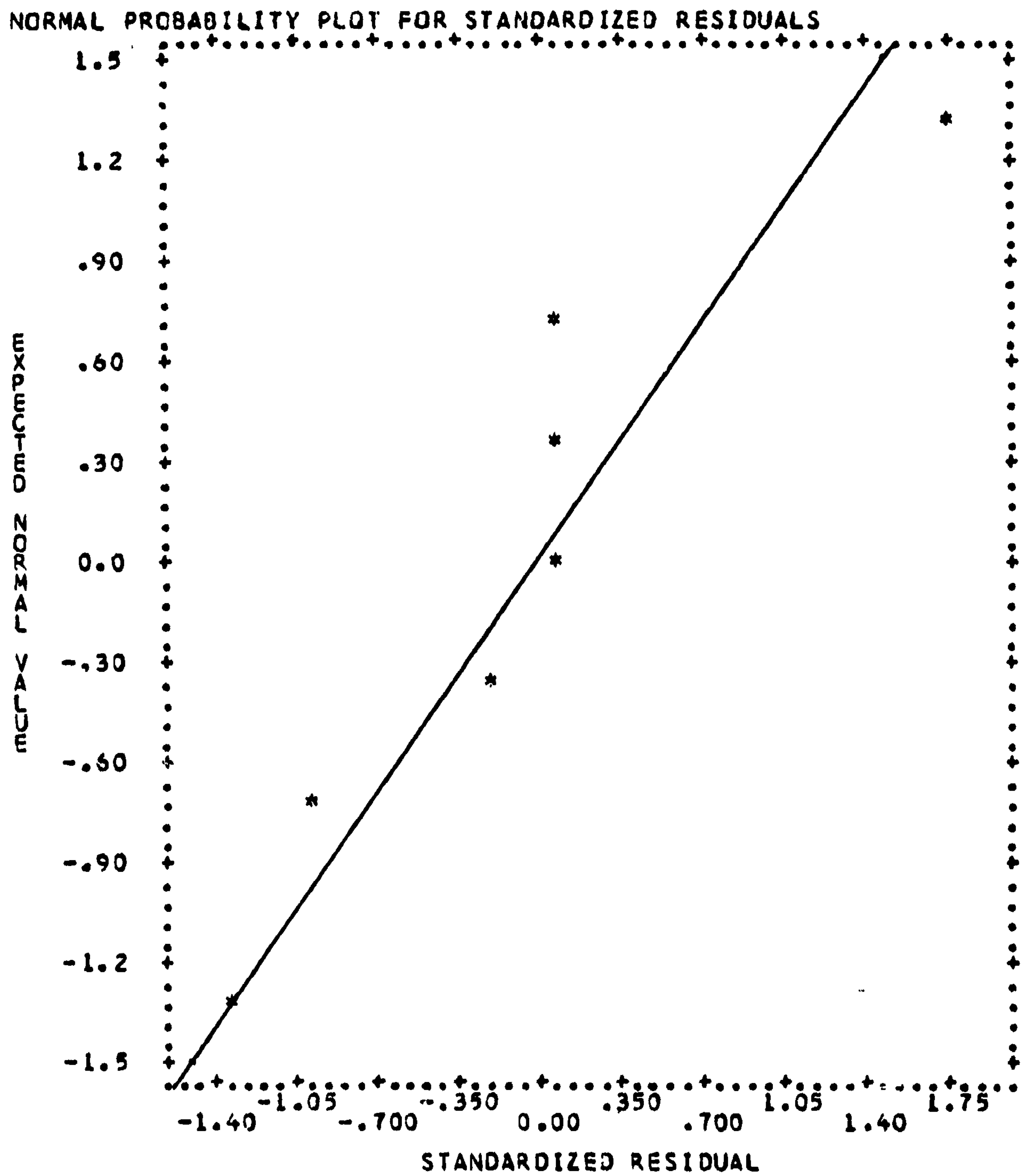


Figure 8
Normal Probability Plot, Group 1

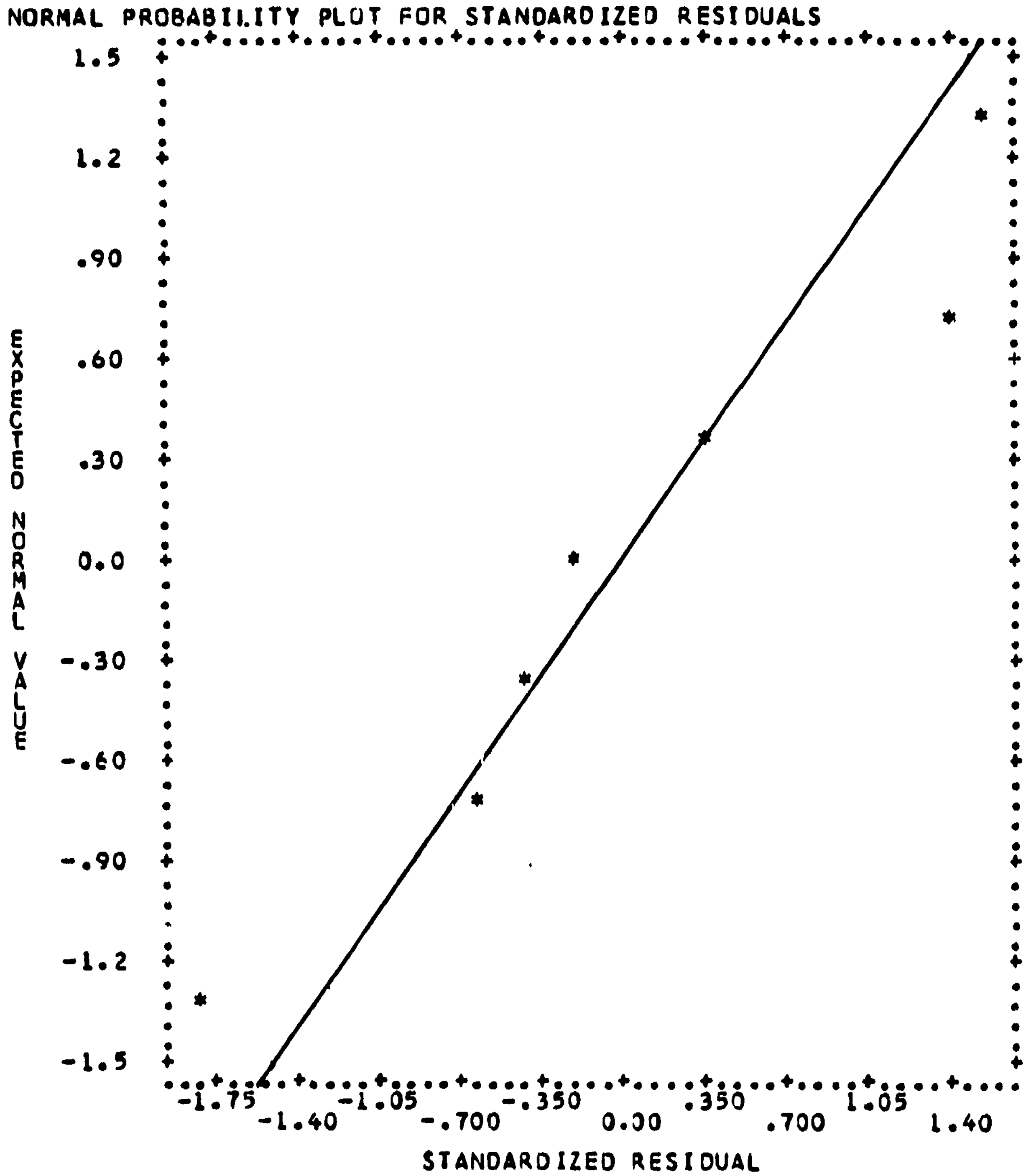


Figure 9
Normal Probability Plot, Group 2

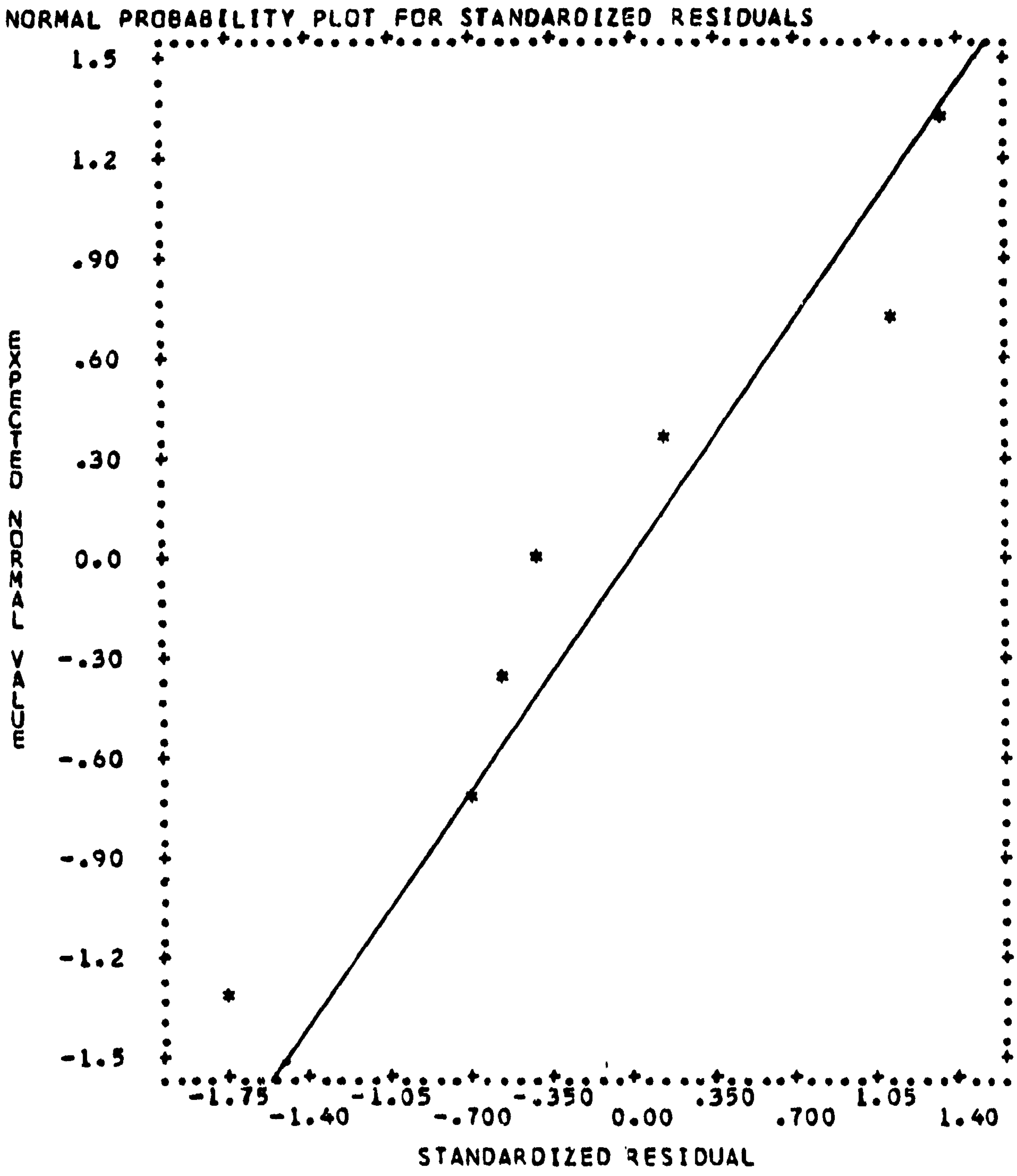


Figure 10

Normal Probability Plot, Group 3

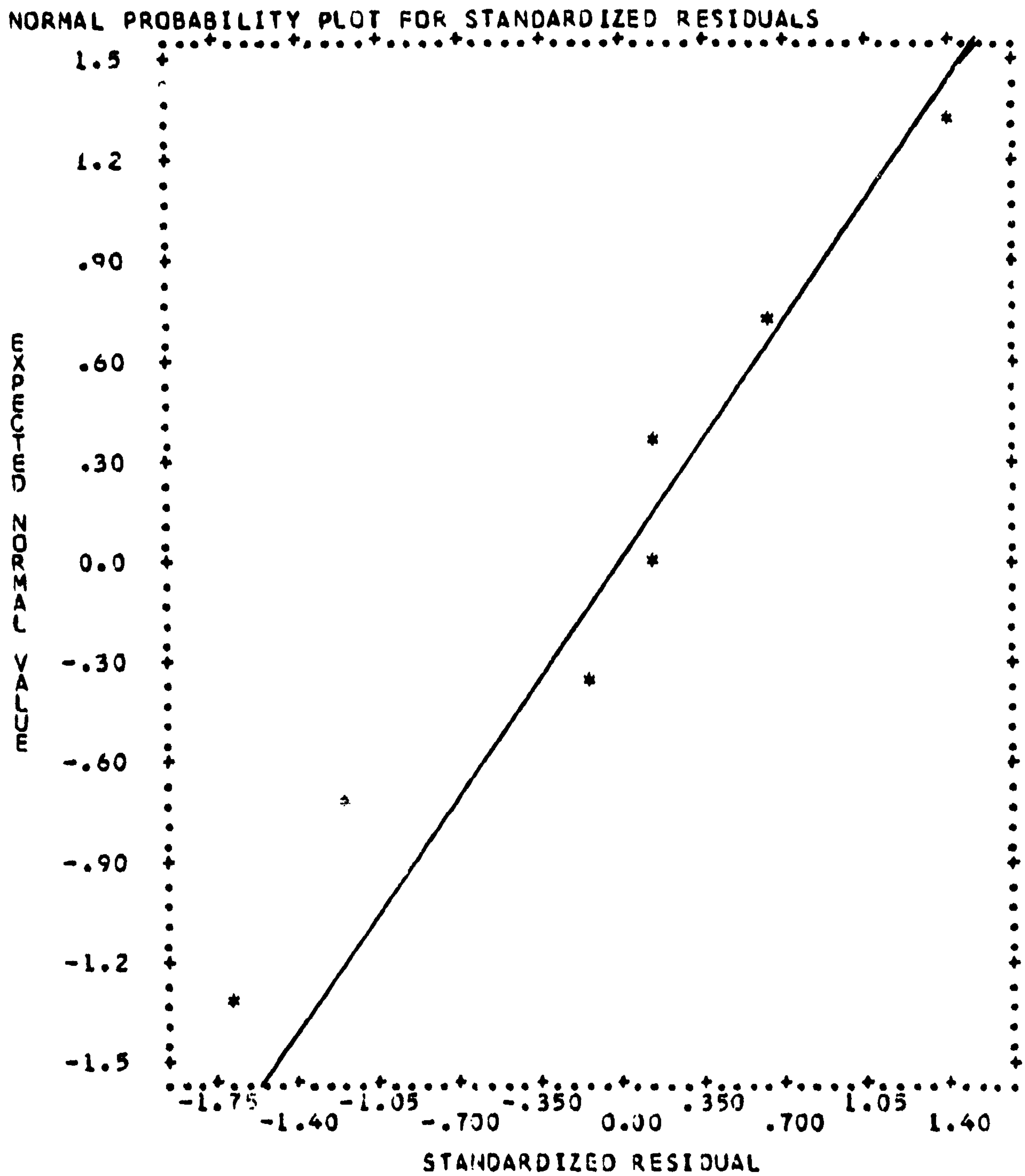


Figure 11

Normal Probability Plot, Group 4

Another interesting question answered by "All Possible Subsets Regression" was: which one of the independent variables gave the most weight to the regression analysis? For candidate model number ((1)), the untransformed data, kill probability, X_1, and maximum range, X_2, gave the highest weight for all four groups, with missile price as the third highest weighted variable. Reaction time, X_3, and average missile speed, X_4, were both removed from the "best" subset in all four groups. So also was X_5 for the expert group (number 3). Out of all possible subsets for all four groups, Group 2 using independent variables X_1, X_2 and X_5 gave the overall best result with an Mallows' $Cp = 2.87$ [8; pg. 532], which is close to the ideal value 3.00. For further details see Table 15.

The same procedure was applied to the data transformed by the best equation using Group 4 as an example. In this case the "best" subset gave a result almost identical to that one obtained by "REGRESS"; see Table 16 for details.

Table 15

Statistics for Bost Subset for Candidate

Model Number ((1))

Group 1:

STATISTICS FOR 'BEST' SUBSET

MALLOWS' CP	2.47
SQUARED MULTIPLE CORRELATION	0.97658
MULTIPLE CORRELATION	0.98822
ADJUSTED SQUARED MULT. CORR.	0.95316
RESIDUAL MEAN SQUARE	0.011002
STANDARD ERROR OF EST.	0.104892
F-STATISTIC	41.70
NUMERATOR DEGREES OF FREEDOM	3
DENOMINATOR DEGREES OF FREEDOM	3
SIGNIFICANCE	0.0060

VARIABLE NO.	NAME	REGRESSION COEFFICIENT	STANDARD ERROR
	INTERCEPT	-0.798151	0.625964
1	X1	2.79295	0.749150
2	X2	-0.0203927	0.00553846
5	X5	0.498094D-06	0.241851D-06

STAND. COEF.	T-STAT.	2TAIL SIG.	TOL-ERANCE	CONTRIBUTION TO R-SQUARED
-1.647	-1.28	0.292		
0.495	3.73	0.034	0.442672	0.108504
-0.424	-3.68	0.035	0.589101	0.105835
0.[illegible]33	2.06	0.132	0.609721	0.033112

THE CONTRIBUTION TO R-SQUARED FOR EACH VARIABLE IS THE AMOUNT BY WHICH R-SQUARED WOULD BE REDUCED IF THAT VARIABLE WERE REMOVED FROM THE REGRESSION EQUATION.

Group 2:

```
STATISTICS FOR 'BEST' SUBSET
MALLOWS' CP                                  2.87
SQUARED MULTIPLE CORRELATION              0.98052
MULTIPLE CORRELATION                      0.99021
ADJUSTED SQUARED MULT. CORR.              0.96105
RESIDUAL MEAN SQUARE                     0.010483
STANDARD ERROR OF EST.                   0.102385
F-STATISTIC                                 50.34
NUMERATOR DEGREES OF FREEDOM                    3
DENOMINATOR DEGREES OF FREEDOM                  3
SIGNIFICANCE                               0.0046
```

VARIABLE NO.	NAME	REGRESSION COEFFICIENT	STANDARD ERROR
	INTERCEPT	-1.03218	0.611004
1	X1	3.00518	0.731245
2	X2	-0.0171122	0.00540609
5	X5	0.758671D-06	0.236071D-06

STAND. COEF.	T-STAT.	2TAIL SIG.	TOL-ERANCE	CONTRIBUTION TO R-SQUARED
-1.990	-1.69	0.190		
0.498	4.11	0.026	0.442672	0.109654
-0.332	-3.17	0.051	0.589101	0.065051
0.332	3.21	0.049	0.609721	0.067055

THE CONTRIBUTION TO R-SQUARED FOR EACH VARIABLE IS THE AMOUNT BY WHICH R-SQUARED WOULD BE REDUCED IF THAT VARIABLE WERE REMOVED FROM THE REGRESSION EQUATION.

Group 3:

```
STATISTICS FOR 'BEST' SUBSET
MALLOWS' CP                                 0.57
SQUARED MULTIPLE CORRELATION             0.96250
MULTIPLE CORRELATION                     0.98107
ADJUSTED SQUARED MULT. CORR.             0.94374
RESIDUAL MEAN SQUARE                    0.010986
STANDARD ERROR OF EST.                  0.104812
F-STATISTIC                                51.33
NUMERATOR DEGREES OF FREEDOM                   2
DENOMINATOR DEGREES OF FREEDOM                 4
SIGNIFICANCE                              0.0014
```

VARIABLE NO. NAME	REGRESSION COEFFICIENT	STANDARD ERROR
INTERCEPT	-0.790969	0.565939
1 X1	2.87949	0.647942
2 X2	-0.0229680	0.00552599

STAND. COEF.	T-STAT.	2TAIL SIG.	TOL-ERANCE	CONTRIBUTION TO R-SQUARED
-1.790	-1.40	0.235		
0.560	4.44	0.011	0.590861	0.185173
-0.524	-4.16	0.014	0.590861	0.161974

THE CONTRIBUTION TO R-SQUARED FOR EACH VARIABLE IS THE AMOUNT BY WHICH R-SQUARED WOULD BE REDUCED IF THAT VARIABLE WERE REMOVED FROM THE REGRESSION EQUATION.

Group 4:

STATISTICS FOR 'BEST' SUBSET	
MALLOWS' CP	2.47
SQUARED MULTIPLE CORRELATION	0.97476
MULTIPLE CORRELATION	0.98730
ADJUSTED SQUARED MULT. CORR.	0.94952
RESIDUAL MEAN SQUARE	0.012192
STANDARD ERROR OF EST.	0.110417
F-STATISTIC	38.62
NUMERATOR DEGREES OF FREEDOM	3
DENOMINATOR DEGREES OF FREEDOM	3
SIGNIFICANCE	0.0068

VARIABLE NO. NAME	REGRESSION COEFFICIENT	STANDARD ERROR
INTERCEPT	-0.696638	0.658937
1 X1	2.69054	0.788612
2 X2	-0.0216604	0.00583020
5 X5	0.518534D-06	0.254591D-06

STAND. COEF.	T-STAT.	2TAIL SIG.	TOL-ERANCE	CONTRIBUTION TO R-SQUARED
-1.418	-1.06	0.368		
0.470	3.41	0.042	0.442672	0.097938
-0.444	-3.72	0.034	0.589101	0.116136
0.239	2.04	0.134	0.609721	0.034904

THE CONTRIBUTION TO R-SQUARED FOR EACH VARIABLE IS THE AMOUNT BY WHICH R-SQUARED WOULD BE REDUCED IF THAT VARIABLE WERE REMOVED FROM THE REGRESSION EQUATION.

Table 16

Statistics for Best Subset for the Best Equation,

Candidate Model Number ((13)), Group 4

STATISTICS FOR 'BEST' SUBSET

SQUARED MULTIPLE CORRELATION	0.99319
MULTIPLE CORRELATION	0.99659
ADJUSTED SQUARED MULT. CORR.	0.98978
RESIDUAL MEAN SQUARE	0.002400
STANDARD ERROR OF EST.	0.048991
F-STATISTIC	291.60
NUMERATOR DEGREES OF FREEDOM	2
DENOMINATOR DEGREES OF FREEDOM	4
SIGNIFICANCE	0.0000

VARIABLE NO.	VARIABLE NAME	REGRESSION COEFFICIENT	STANDARD ERROR
	INTERCEPT	-1.84625	0.217266
1	X1NEW	2.35347	0.0986511
2	X2NEW	0.484809	0.0590142

STAND. COEF.	T-STAT.	2TAIL SIG.	TOL-ERANCE	CONTRIBUTION TO R-SQUARED
-3.809	-8.50	0.001		
1.124	23.86	0.000	0.767553	0.969220
0.387	8.22	0.001	0.767553	0.114931

THE CONTRIBUTION TO R-SQUARED FOR EACH VARIABLE IS THE AMOUNT BY WHICH R-SQUARED WOULD BE REDUCED IF THAT VARIABLE WERE REMOVED FROM THE REGRESSION EQUATION.

LIST OF REFERENCES

1. Raisbeck, Gordon, "How the Choice of Measures of Effectiveness Constrains Operational Analysis", Interfaces, Vol. 9, no. 4, August 1979.

2. Jones, K.M., and others, "Judgement Modeling for Effective Policy and Decision Making", U.S. Air Force Office of Scientific Research, August 1975.

3. Mosteller, F. and Tukey, J.W., "Data Analysis and Regression", Addison Wesley, 1977.

4. Lindsay, Glenn F., "Scaling with the Constant Sum Method", Naval Postgraduate School, February, 1980.

5. Torgersen, S.W., "Theory and Methods of Scaling", John Wiley & Sons, Inc., New York, 1967.

6. Richards, F. Russel, "A User's Guide to the OA3660 APL Workspace", Naval Postgraduate School, October, 1978.

7. Dixon, W.F., and others, "Bimedical Computer Programs", University of California Press, 1977.

8. Hocking, R.R. and Leslie, R.N., "Selection of the Best Subset in Regression Analysis", Technometrics, Vol. 9, No. 4, November, 1967.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, California 93940	2
3.	Department Chairman, Code 55 Department of Operations Research Naval Postgraduate School Monterey, California 93940	1
4.	Assoc. Professor Glenn F. Lindsay, Code 55Ls Department of Operations Research Naval Postgraduate School Monterey, California 93940	2
5.	Assoc. Professor Alvin F. Andrus, Code 55As Department of Operations Research Naval Postgraduate School Monterey, California 93940	1
6.	Mr. Don Vance U.S. Army Air Defense School ATSA-CDF-S Fort Bliss, Texas 79916	1
7.	Chief of Naval Operations/Royal Norwegian Navy Oslo Mil Huseby Oslo 1 Norway	3
8.	The Commandant Royal Norwegian Naval Academy P.O. Box 25 5034 Ytre Laksevaag Norway	2
9.	The Commandant KNM Tordensjold 5078 Haakonsvern Norway	2

10. The Commandant 1
Royal Norwegian Air Force Academy
Trondheim Mil
7000 Trondheim
Norway

11. LCDR Knut O. Flaathen 1
Neptunv. 6
5071 Loddefjord
Norway